Class : JEE Main 2015
Subject : Physics
Paper Code : D

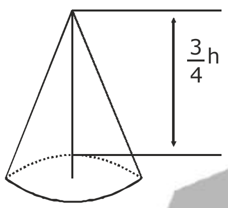
1. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to :

1. $\frac{5th}{8}$ 2. $\frac{3h^2}{8R}$

3. $\frac{h^2}{4R}$ 4. $\frac{3h}{4}$

Solution: (4)

$$y_{cm} = \frac{\int y_{dm}}{m}$$



2. A red LED emits light at 0.1 watt uniformly around it The amplitude of the electric field of the light at a distance of 1m from the diode is

1. 5.48 V/m 2. 7.75 V/m

3. 1.73 V/m 4. 2.45 V/m

Solution: (4)

Intensity of EM wave (I) = UC

$$= \varepsilon_o E^2 C = \frac{\varepsilon_o E_p^{\ 2} C}{2} \quad \ldots (1)$$

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{A} \quad \ldots (2)$$

From (1) and (2)

$$E_p^{\ 2} = \frac{2P}{4\pi r^2 \varepsilon_o C} = 6 \Rightarrow E_p = \sqrt{6} = 4.245 \text{ v/m}$$

3. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, thetime period changes to T_M. Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:

(g = gravitational acceleration)

1. $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ 2. $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

3. $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ 4. $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

Solution: (3)

$$Y = \frac{F/A}{\Delta\ell/\ell} = \frac{F\ell}{A\left(\ell^1-\ell\right)} \Rightarrow \frac{1}{Y} = \left[\frac{\ell^1-\ell}{\ell}\right]\frac{A}{F}$$

Extra Force F = mg

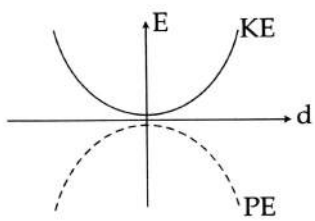
$T = 2\pi\sqrt{\ell/g}$ and $T_m = 2\pi\sqrt{\ell^1/g}$

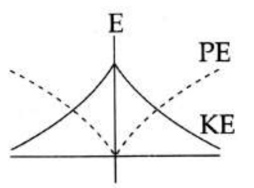
By solving $\frac{\ell^1-\ell}{\ell} = \left(\frac{T_m}{T}\right)^2 - 1$

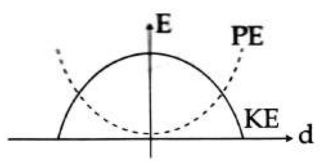
$$\therefore \frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{mg}$$

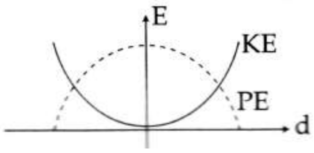
4 For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?

(graphs are schematic and not drawn to scale)

1. 

2. 

3. 

4. 

Solution: (4)

$KE = \frac{1}{2}\, mw^2\,(A^2 - x^2)$ — Inverted Parabola

$PE = \frac{1}{2}\, mw^2 x^2$ — Parabola

5. A train is moving on a straight track with speed 20 ms-1. It is blowing its whistle at the frequency of 100 Hz. The precentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to:

1. 18% 2. 24%
3. 6% 4. 12%

Solution: (4)

Observer at rest and source is moving.
$c = 320$ m/s; $V_s = 20$m/s; $V_0 = 1000\ H_3$

$$V_i = \frac{c}{c - V_s}\ ;\ V_f = \frac{c}{c + V_s}$$

$$\%\ \text{change} = \frac{V_i - V_f}{V_i} \times 100 = \frac{2V_S}{C + V_S} \times 100$$

$\approx 12\%$

6. when 5 v potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5x10-4 ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to :

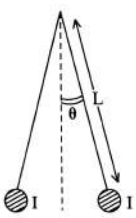
1. $1.6 \times 10^{-6}\ \Omega$m 2. $1.6 \times 10^{-5}\ \Omega$m
2. $1.6 \times 10^{-8}\ \Omega$m 3. $1.6 \times 10^{-7}\ \Omega$m

Solution: (2)

Ohm's law in microscopic form $E = J\rho$

$$\Rightarrow \rho = \frac{E}{J} = \frac{V/\ell}{neV_d}$$

Putting Values $\rho \approx 1.6 \times 10^{-5}\ \Omega$m

7. 

Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of is :

(g = gravitational acceleration)

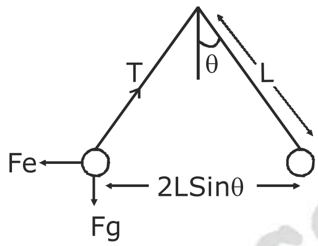
1. $2\sqrt{\frac{\pi g L}{\mu_0}\tan\theta}$ 2. $\sqrt{\frac{\pi \lambda g L}{\mu_0}\tan\theta}$

3. $\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$ 4. $2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$

Solution: (4)

$$\tan\theta = \frac{Fe}{Fg}$$

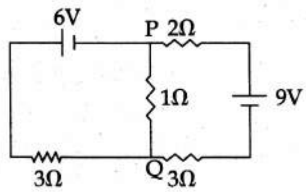
$$\text{Fe per unit length} = \frac{U_0 I^2}{2\pi\, 2L\,\text{Sin}\theta}$$



Fg per unit length $= \lambda g$

$$\Rightarrow \tan\theta = \frac{\mu_0 I^2}{4\pi L \sin\theta \lambda g}$$

$$\Rightarrow I = 2\sin\theta\sqrt{\frac{\pi\lambda L g}{\mu_0\cos\theta}}$$

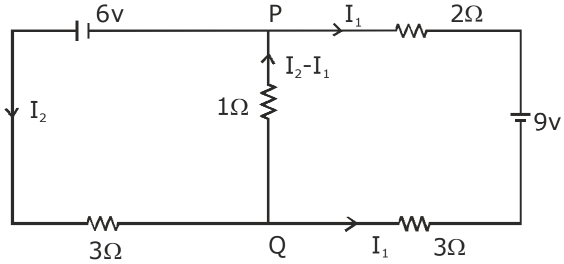
8. 

In the circuit shown, the current in the 1Ω resistor is:

1. 0.13 A, from Q to P
2. 0.13 A, from P to Q
3. 1.3 A, from P to Q
4. 0A

Solution: (1)

Using KVL
$6 = 4I_2 - I_1$ — (1)
$9 = 6I_1 - I_2$ — (2)



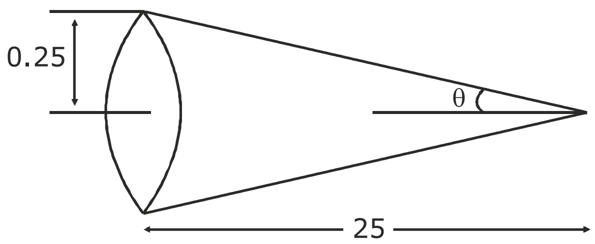
Solving $I_1 = \frac{42}{23}$, $I_2 = \frac{45}{23}$

$\therefore I_2 - I_1 = \frac{3}{23} \approx 0.13$ A from Q to P

9. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25cm, the minimum sepration between two objects that human eye can resolve at 500 nm wavelength is:

1. 100μm 2. 300μm
3. 1μm 4. 30μm

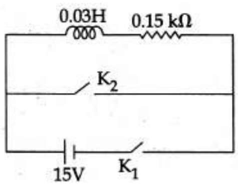
Solution: (4)



Resolving Power = $\frac{1.22\lambda}{2\mu \sin\theta}$

$= 3.05 \times 10^{-5} \approx 30\ \mu m$

10. An inductor (L=0.03H) and resistor (R=0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0 k_1 is is opened and key k_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be : ($e^5 \cong 150$)

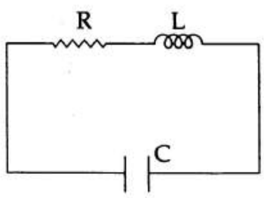


1. 6.7 mA 2. 0.67 mA
3. 100 mA 4. 67 mA

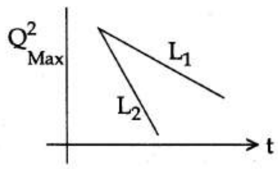
Solution: (2)

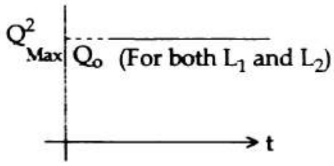
$I = I_0\, e^{-t/\tau} = \frac{1}{10} e^{-5} = \frac{1}{1500} \approx 0.67$ mA

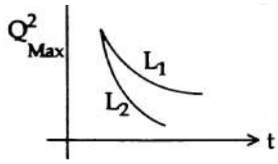
11. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below:

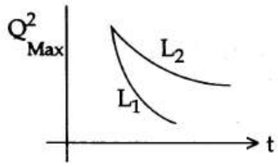


If student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_2 > L_2$) of L then which of the following represents this graph correctly? (plots are schematic and not drawn toscale)

1. 

2. 

3. 

4. 

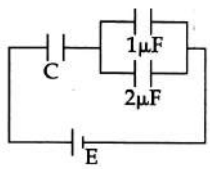
Solution: (3)

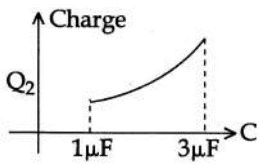
$Q_{max} = Q_0 e^{-kt}$

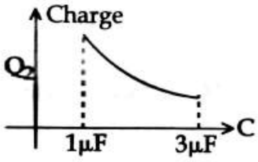
$K \propto \frac{1}{L}$

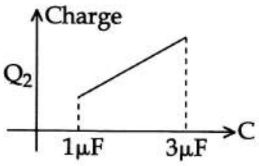
Correct option can selected by looking the options.

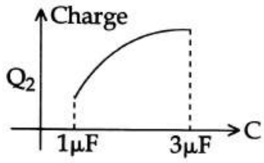
12. In the given circuit, charge Q_2 on 2μF to 3μF. Q_2 as function of 'C' is given properly by : (figures are drawn schematically and are not to scale)



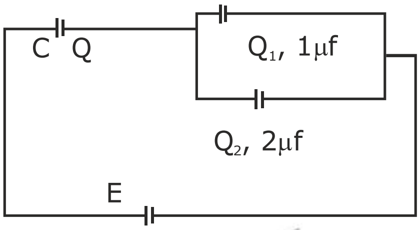
1. 

2. 

3. 

4. 

Solution: (4)



$$\frac{Q_1}{1} = \frac{Q_2}{2} \Rightarrow Q_1 = \frac{Q_2}{2}$$

$$Q = Q_1 + Q_2 = \frac{3Q_2}{2}$$

Total Capacitance of ckt = $\frac{3c}{c+3}$

$$\therefore Q = \frac{3CE}{C+3} = \frac{3Q_2}{2} \Rightarrow Q_2 = \frac{2CE}{C+3}$$

$$\Rightarrow Q_2 \;\alpha\; \frac{C}{C+3}$$

13. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and prependicular to one of its faces is:

1. $\frac{4MR^2}{9\sqrt{3}\pi}$ 2. $\frac{4MR^2}{3\sqrt{3}\pi}$

3. $\frac{MR^2}{32\sqrt{2}\pi}$ 4. $\frac{MR^2}{16\sqrt{2}\pi}$

Solution: (1)

Side of cube (a) $= \frac{2R}{\sqrt{3}}$

Mass of cube (M_c) $= \frac{M \text{ x vol. of cube}}{\text{vol. of sphere}} = \frac{2M}{\sqrt{3}\pi}$

I_{CM} (for cube) $= \frac{M_c a^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$

14. The period of oscillation of a simple pendulum is T = $2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resoulation. The accuracy in thedetermination of g is:

1. 1% 2. 5%

3. 2% 4. 3%

Solution: (4)

Rearranging g = $\frac{T^2}{4\pi^2 e}$

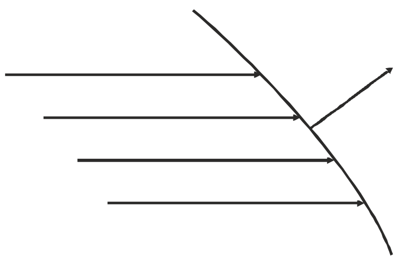
$$\Rightarrow \frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta \ell}{\ell} = \frac{2 \text{ x } 1}{90} + \frac{0.1}{20} \approx 0.03$$

= 3%

15. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Hugyens' principle leads us to conclude that as it travels, the light beam:

1. bends downwards
2. bends upwards
3. becomes narrower
4. goes horizontally without any deflection

Solution: (2)



Velocity of rays away from the ground will be less as reprochive index is more. This will deform the wave front resulting into bending of the beam upwards.

16. A single of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the result signal is / are:

1. 2005 kHz, 200 kHz and 1995 kHz
2. 2000 kHz and 1995 kHz
3. 2 MHz only
4. 2005 kHz and 1995 kHz

Solution: (1)

Frequencies of resultant signal are
= Fc + Fs , Fc, Fc - Fs
= 2005 KHz, 2000 KHz and 1995 KHz

17. A solid body of constant heat capacity 1J/°C is being heated by keeping it in contact with reservoris in two ways.

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is

1. ln2, 2ln2   2. 2ln2, 8ln2

3. ln2, 4ln2   4. ln2, ln2

Solution : (2)

18. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume u=$\frac{U}{V} \propto T^4$ and pressure $P=\frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between t and R is:

1. $T \propto \frac{1}{R}$ 2. $T \propto \frac{1}{R^3}$

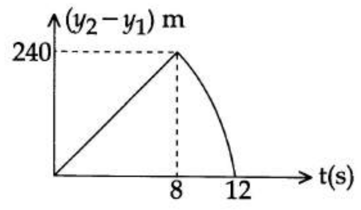
3. $T \propto e^{-R}$ 4. $T \propto e^{-3R}$

Solution : (1)

19. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2)

(*The figures are schematic and not drawn to scale)*

1.


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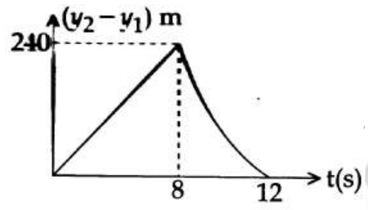

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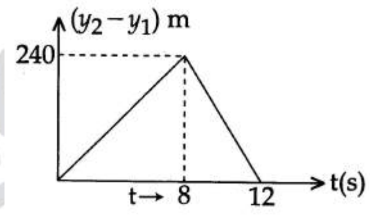

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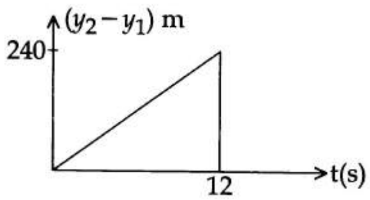


Solution : (1)

Till both stone are in air, they will have constant relative velocity of 30m/s.
$\Delta y = y_2 - y_1 = 30t$
First stone will hit ground after 8 sec
($\because -240 = 10t - 2t^2$).
After 8s, the relative velocity will be aqual to velocity of 2nd stone and the Δx -t graph will be inverted parabola.

20. A uniformly chaged solid sphere of radius R has potential V_0 (measured with respect to ∞)on its surface. For this sphere the equipotential surfaces wityh potentials

$\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

1. $R_1 = 0$ and $R_2 < (R_4 - R_3)$
2. $2R < R_4$
3. $R_1 = 0$ and $R_2 > (R_4 - R_3)$
4. $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

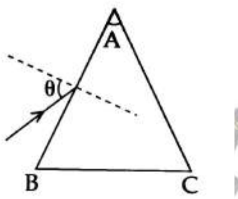
Solution: (1),(2)

For uniformly changed solid sphere

$$V = \begin{cases} \dfrac{Kq}{r} & \text{for}\,(r \ge R) \\ \dfrac{Kq}{2R}\left[3 - \dfrac{r^2}{R^2}\right] & \text{for}\,(r < R) \end{cases}$$

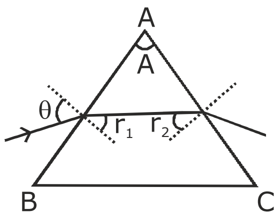
Using fornula's $R_1 = 0$; $R_2 = R/\sqrt{2}$;
$R_3 = 4R/3$; $R_4 = 4R$
After checking the options, (1) and (2) satisfy.

21. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is ix, a ray, incident at an angle 0, on the face AB would get transmitted through the face AC of the prism provided :



1. $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
2. $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
3. $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
4. $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

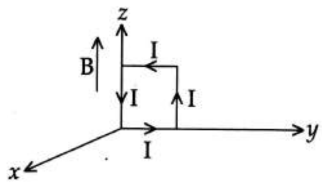
Solution: (3)

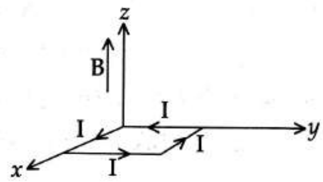


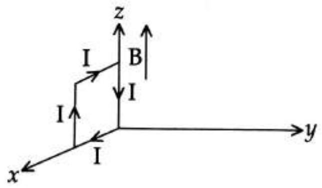
T avoid T_1 R at face AC

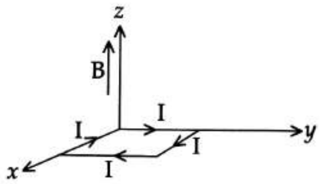
$r_2 \;\alpha \sin^{-1}(1/\mu)$
for prism $A = r_1 + r_2$
$\Rightarrow A - r_1 < \sin^{-1}(1/\mu)$
$\Rightarrow r_1 > A - \sin^{-1}(1/\mu)$
For face AB, $\sin\theta = \mu \sin r_1$
$\Rightarrow \theta = \sin^{-1}(\mu \sin r_1)$
$\Rightarrow \theta > \sin^{-1}(\mu \sin(A - \sin^{-1}(1/\mu)))$

22. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below :

1. 

2. 

3. 

4. 

If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium ?

1. (b) and (d), respectively
2. (b) and (c), respectively
3. (a) and (b), respectively
4. (a) and (c), respectively

Solution: (1)

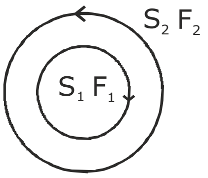
\# For stable equilibrium - magnetic dipole moment and magnetic field should be in same direction. (figure (b))

\# For unstable equilibrium - $\vec{m}$ and $\vec{b}$ should be anti-parallel. (figure (d))

23. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then :

1. $\vec{F_1}$ is radially inwards and $\vec{F_2}=0$
2. $\vec{F_1}$ is radially outwards and $\vec{F_2}=0$
3. $\vec{F_1} = \vec{F_2}=0$
4. $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards

Solution: (3)



Consider the front view.

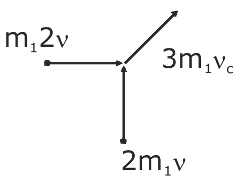
\# The net force F_1 will be zero as it will be in all directions

\# There will be no field outside the solenoid due to S_1. Therefore the force $\vec{F_2} = 0$

24. A Particle of mass 2m moving in the x direction with speed 2v. is hit by another particle of mass 2 m moving in the y direction with speed v. If the collisioin is perfectly inelastic, the percentage loss in the energy during the collision is close to:

1. 56%
2. 62%
3. 44%
4. 50%

Solution: (1)



Using conservation of momentum

$$3m\vec{v}_c = 2mv\left(\hat{i}+\hat{j}\right) \Rightarrow \left|\vec{v}_c\right| = \frac{2\sqrt{2}}{3}v$$

$KE_{loss} = KE_i - KE_f$

$$= 3mv^2 - \frac{1}{2}.\,3m\left(\frac{2\sqrt{3}v}{3}\right)^2$$

$$= \frac{5}{3}mv^2$$

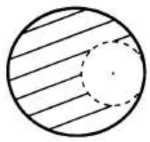
25. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is

$$\left(\gamma = \frac{C_P}{C_V}\right)$$

1. $\frac{\gamma+1}{2}$
2. $\frac{\gamma-1}{2}$
3. $\frac{3\gamma+5}{6}$
4. $\frac{3\gamma-5}{6}$

26. From a solid sphere of mass M and radius R, a spherical portion of radius is $\frac{R}{2}$ removed, as shown in the figure. Taking gravitational potential V=0 at r=∞ the potential at the centre of the cavity thus formed is :

(G = gravitational constant)



1. $\frac{-2GM}{3R}$
2. $\frac{-2GM}{R}$
3. $\frac{-GM}{2R}$
4. $\frac{-GM}{R}$

Solution: (4)

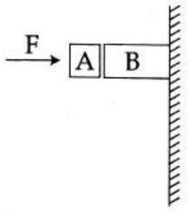
$V = V_s - V_c$

$$V_s = -\frac{GM}{2R}\left[3 - \frac{\left(R/2\right)^2}{R^2}\right]$$

$$= -\frac{11}{8}\frac{GM}{R}$$

$$V_c = -\frac{3GM^1}{2r} = -\frac{3G(M/8)}{2(R/2)} = -\frac{3}{8}\frac{GM}{R}$$

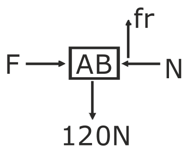
$$V = V_S - V_C = -\frac{GM}{R}$$

27. 

Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :

1. 120 N
2. 150 N
3. 100 N
4. 80 N

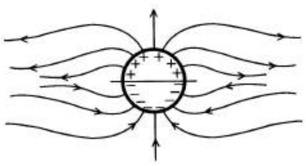
Solution: (1)

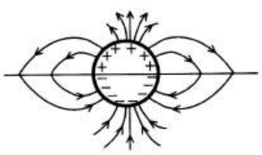


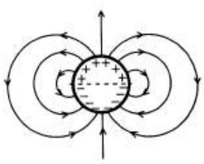
For equilibrium F_r = 120N

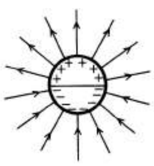
28. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in :

(figures are schematic and not drawn to scale)

1. 

2. 

3. 

4. 

Solution: (3)

We can treat the given configration as an electric dipole.

29. As an electron makes a transition from an excited state of a hydrogen – like atom/ion:

1. kinetic energy decreases, potential energy increases but total energy remains same
2. kinetic energy and total energy decrease but potential energy increases
3. its kinetic energy increases but potential energy and total energy decrease
4. kinetic energy, potential energy and total energy decrease

Solution: (3)

$KE = -TE = -\frac{PE}{2}$

$KE \ \alpha \ \frac{1}{n^2}$, KE $\uparrow$ as n $\downarrow$

$PE \ \alpha \ \frac{-2}{n^2}$, PE $\downarrow$ as n $\downarrow$

$TE \ \alpha \ \frac{-1}{n^2}$, TE $\downarrow$ as n $\downarrow$

30. Match **List - I** (Fundamental Experiment) with **List - II** (its conclusion) and select the correct option from the choices given below the list :

	List - I		List - II
(A)	Franck-Hertz Experiment.	(i)	Particle nature of light
(B)	Photo-electric experiment.	(ii)	Discrete energy levels of atom
(C)	Davison - Germer Experiment.	(iii)	Wave nature of electron
		(iv)	Structure of atom

1. (A)- (ii) (B)-(i) (C)-(iii)
2. (A)- (iv) (B)-(iii) (C)-(ii)
3. (A)- (i) (B)-(iv) (C)-(iii)
4. (A)- (ii) (B)-(iv) (C)-(iii)

Solution: (1)