31. Let $\vec{a}, \vec{b}$ *and* $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{2}|\vec{b}||\vec{c}||\vec{a}|$. If θ is the angle between vectors $\vec{b}$ *and* $\vec{c}$, then a value of $\sin\theta$ is

1. $\frac{2}{3}$ 2. $\frac{-2\sqrt{3}}{3}$

3. $\frac{2\sqrt{2}}{3}$ 4. $\frac{-\sqrt{2}}{3}$

Solution: (3)

$$(\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a} = \frac{1}{3}(|\vec{b}||\vec{c}|)\vec{a}$$

comparing $\cos\theta = -\frac{1}{3} \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$

32. Let O be the vertex and Q be any point on parabola, x^2=8y. if the point P divides the line segment OQ internally in the ratio 1:3, then the locus of P is :

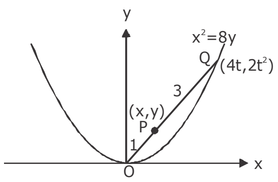
1. $y^2 = 2x$ 2. $x^2 = 2y$

3. $x^2 = y$ 4. $y^2 = x$

Solution: (2)

$$x = \frac{4t}{4} = t$$

$$y = \frac{2t^2}{4} = \frac{t^2}{2}$$



so, $x^2 = 2y$ is locus of p.

33. If the angles of elevation of the top of a tower from three collinear points, A,B, and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio,

AB : BC, is :

1. $1:\sqrt{3}$ 2. 2:3

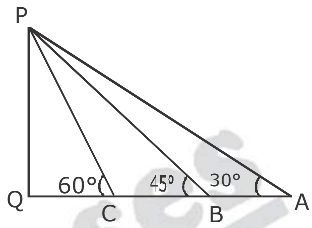
3. $\sqrt{3}:1$ 4. $\sqrt{3}:\sqrt{2}$

Solution: (3)

$$\frac{AQ}{\sqrt{3}} = BQ = CQ\sqrt{3}$$

$\therefore$ AQ =3CQ, AC = 2CQ

BC = $(\sqrt{3}-1)CQ$



AB = $(3-\sqrt{3})CQ = \sqrt{3}(\sqrt{3}-1)CQ$

$\therefore$ AB : BC = $\frac{\sqrt{3}(\sqrt{3}-1)CQ}{(\sqrt{3}-1)CQ} = \frac{\sqrt{3}}{1}$

34. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0,0), (0, 41) and (41,0), is:

1. 820 2. 780

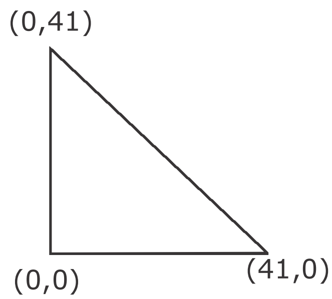
3. 901 4. 861

Solution: (2)

number of points

= 1+2+3+ .... + 39

$$= \frac{39 \times 40}{2} = 780$$



35. the equation of the plane containing the line 2x - 5y + z = 3; x + y + 4z = 5, and parallel to the plane, x + 3y + 6z = 1 is :

1. x + 3y + 6z = 7
2. 2x + 6y + 12z = -13
3. 2x + 6y + 12z = 13
4. x + 3y + 6z = -7

Solution: (1)

equation of required plane is

$2x - 5y + z - 3 + \lambda(x + y + 4z - 5) = 0$

$\Rightarrow (2+\lambda)x + (\lambda - 5)y + (1 - 4\lambda)z - 3 - 5\lambda = 0$.......(1)

Since, Required plane is parallel to

$x + 3y + 6z = 1$,

So , $\frac{2+\lambda}{1} = \frac{\lambda-5}{3} = \frac{1-4\lambda}{6} = \frac{3+5\lambda}{1}$.

Solving any two pair, we get $\lambda = 11/2$.

Putting value of λ in equation (1), we get

x + 3y + 6z = 7

36. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A x B, each having at least three elements is:

1. 275
2. 510
3. 219
4. 256

Solution: (3)

number of subsets =

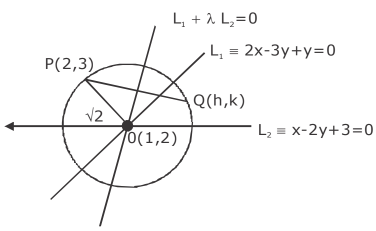
$2^8 - \{c(8,0)+c(8,1)+c(8,2)\} = 256 - (1+8+28)$

= 219.

37. Locus of the image of the point (2,3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \varepsilon R$, is a:

1. circle of radius $\sqrt{2}$.
2. circle of radius $\sqrt{3}$.
3. straight line parallel to x-axis.
4. straight line parallel to y-axis.

Solution: (1)



Clearly, the locus of Q is a circle.

38. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to:

1. 2
2. $\frac{1}{2}$
3. 4
4. 3

Solution: (1)

$$\lim_{x\to 0} \frac{(1-\cos 2x)}{x^2} \times \frac{(3+\cos x)}{\frac{\tan 4x}{x}}$$

$$= \frac{4}{2} \times \frac{4}{2} = 2$$

39. The distance of the point (1,0,2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$ is:

1. $3\sqrt{21}$
2. 13
3. $2\sqrt{14}$
4. 8

Solution: (2)

Equation of line is $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda(\text{say})$

$\Rightarrow x = 3\lambda + 2,\ y = 4\lambda - 1,\ z = 12\lambda + 2$

Putting these values in plane x - y + z = 16,

we get , $\lambda = 1$.

So,point of intersection (5,3,14)

distance = $\sqrt{4^2 + 3^2 + 12^2}$, = 13

40. The sum of coefficients of integral powers of x in the bionomial expansion of $(1 - 2\sqrt{x})^{50}$ is

1. $\frac{1}{2}(3^{50} - 1)$ is
2. $\frac{1}{2}(2^{50} + 1)$
3. $\frac{1}{2}(3^{50} + 1)$
4. $\frac{1}{2}(3^{50})$

Solution: (3)

Let $\sqrt{x} = y$.

Now,

$(1-2y)^{50} = a_0 + a_1 y + a_2 y^2 + \ldots . a_{50} y^{50}$ ......(1)

Putting y = 1, in (1) we get

$1 = a_0 + a_1 + a_2 + \ldots . a_{50}$ .....(2)

putting y = -1,in (1) we get,

$3^{50} = a_0 - a_1 + a_2 - a_3 \ldots . + a_{50}$ .....(3)

Adding (2) and (3), we get

$3^{50} + 1 = 2(a_0 + a_2 + \ldots . + a_{50})$

$\Rightarrow (a_0 + a_2 + \ldots . + a_{50}) = \dfrac{3^{50}+1}{2}$

41. the sum of first 9 terms of the series

$\dfrac{1^3}{1} + \dfrac{1^3+2^3}{1+3} + \dfrac{1^3+2^3+3^3}{1+3+5} +$ ..... is:

1. 14
2. 192
3. 71
4. 96

Solution: (4)

$T_n = \dfrac{(n+1)^2}{4} = \dfrac{(n^2+2n+1)}{4}$

$s_n = \sum Tn = \dfrac{1}{9}\left[\dfrac{n(n+1)(2n+1}{6} + 2\dfrac{(n)(n+1)}{2} + n\right]$

$= \dfrac{n}{24}(2n^2 + 9n + 13)$

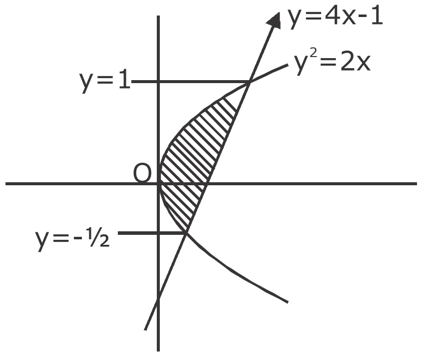
$\therefore s_q = \dfrac{9}{24} \times (2 \times 81 + 81 + 13) = 96$

42. The area (in sq. units) of the region described by

$\{(x,y) : y^2 \leq 2x$ and $y \geq 4x-1\}$ is:

1. $\dfrac{15}{64}$
2. $\dfrac{9}{32}$
3. $\dfrac{7}{32}$
4. $\dfrac{5}{64}$

Solution: (2)



Area $= \displaystyle\int_{y=-1/2}^{y=1} \left(\frac{y+1}{4} - \frac{y^2}{2}\right) dy = 9/32$

43. The set of all values of λ for which the system of linear equations:

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_1$

$-x_1 + 2x_3 = \lambda x_3$

has a non-trivial solution,

1. contains two elements
2. contains more than two elements.
3. is an empty set
4. is singleton.

Solution : (1)

$(2 - \lambda)\, x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3 + \lambda)\, x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$\Rightarrow (\lambda - 1)^2(\lambda + 3) = 0$

$\Rightarrow \lambda = 1$ and $\lambda = -3$.

So, set contains two elements.

44. a complex number z is said to be unimodular if $|z|=1$. Suppose z_1 and z_2 are complex number such that $\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

1. circle of radius 2.
2. circle of radius $\sqrt{2}$.
3. straight line parallel to x-axis.
4. straight line parallel to y-axis.

Solution : (1)

$\left|\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1 \Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

$\Rightarrow (z_1 - 2z_2)\left(\overline{z_1 - 2z_2}\right) = (2 - z_1\bar{z}_2)\left(\overline{2 - z_1\bar{z}_2}\right)$

$\Rightarrow (|z_1|^2 - 4)(1 - |z_2|^2) = 0$

$\Rightarrow |z_1|^2 = 4$, $\because z_2$ is not unimodular

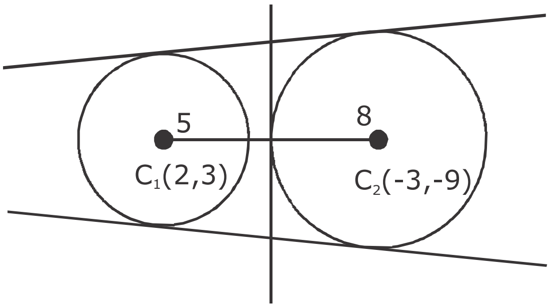
$\Rightarrow |z_1| = 2$ which lies on a circle of radius 2.

45. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$

and $x^2 + y^2 + 6x + 18y + 26 = 0$ is :

1. 3
2. 4
3. 1
4. 2

Solution : (1)



$x^2 + y^2 - 4x - 6y - 12 = 0$

$C_1(2,3)$, $r_1 = \sqrt{4+9+12} = 5$

$x^2 + y^2 + 6x + 18y + 26 = 0$

$C_2 = (-3,-9)$, $r_2 = \sqrt{9+81-26} = 8$

$\overline{C_1C_2} = \sqrt{25+144} = 13$

46. The number of integers greater than 6,000 that can be formed, using the digits, 3,5,6, 7 and 8, without repetition, is:

1. 120
2. 72
3. 216
4. 192

Solution: (4)

Total number of integers = 3(24) + 5!

= 72 + 120 = 192.

47. Let y(x) be the solution of the differential equation

$(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1)$.

Then y(e) is equal to:

1. 2
2. 2e
3. e
4. 0

Solution:

The question is not theoratically correct.

48. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -1 \\ a & 2 & b \end{bmatrix}$ is a martix satisfying the equation $AA^T = 9I$, where I is 3 x 3 identity matrix, then the ordered pair (a,b) is equal to:

1. (2,1)
2. (-2, -1)
3. (2,-1)
4. (-2,1)

Solution:(2)

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 9 & 2 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 9 \\ 2 & 1 & 2 \\ 2 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Equating the corresponding elements, we get,

a +2b = -4 and a - b = -1.

Solving both equations,we get

a = -2 and b = -1.

49. If m is the A.M. of two distinct real numbers l and n (l,n>1) and G_1, G_2 and G_3 are three geomertic means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.

1. 4 lmn^2
2. 4 $l^2m^2n^2$
3. 4 l^2mn
4. 4 lm^2n

Solution: (4)

l + n = 2m.

l,G_1,G_2,G_3,n are in G.P.

$\Rightarrow$ common ratio = r = $\left(\frac{n}{l}\right)^{1/4}$

$\Rightarrow G_1^4 = l^3n, G_2^4 = l^2n^2, G_3^4 = ln^3$

$\Rightarrow G_1^4 + 2G_2^4 + G_3^4 = l^3n + 2l^2n^2 + ln^3$

$= ln(l+n)^2 = ln(4m^2) = 4lm^2n$.

50. the negation of $\sim s \wedge (\sim r \vee s)$ is equivalent to:

1. $s \wedge (r \vee \sim s)$
2. $s \wedge r$
3. $s \wedge \sim r$
4. $s \wedge (r \wedge \sim s)$

Solution:(2)

$\sim[\sim s \vee (\sim r \wedge s)] = \sim((\sim s \vee \sim r) \wedge (\sim s \vee s))$

$= \sim(\sim s \vee \sim r) = (s \wedge r)$.

51. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals:

1. $-(x^4+1)^{\frac{1}{4}}+c$ 2. $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

3. $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ 4. $(x^4+1)^{\frac{1}{4}}+c$

Solution:(2)

$$\int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^5(1+\frac{1}{x^4})^{3/4}}$$

$$= \int \frac{(-t^3)dt}{(t)^3} \left\{\text{put } 1+\frac{1}{x^4} = t^4 \Rightarrow \frac{1}{x^5}dx = -t^3\,dt\right.$$

$$= -t + C = -\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$$

52. The normal to the curve, $x^2+2xy-3y^2=0$, at (1,1):

1. meets the curve again in the third quadrant.
2. meets the curve again in the fourth quadrant.
3. does not meet the curve again.
4. meets the curve again in the second quadrant.

Solution:(2)

$x^2 + 2xy - 3y^2 = 0 \Rightarrow (x - y)(x + 3y) = 0$

$\Rightarrow$ x= y or x + 3y = 0.

(1,1) lies on the line x = y.

Equation of normal to it is y - 1 = (- 1)(x - 1)

$\Rightarrow$ x + y = 2.

This nomal meets the curve x + 3y = 0 at (3,-1) which lies in 4th quadrant.

53. Let

$$\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

where $|x| < \frac{1}{\sqrt{3}}$. Then a value to y is:

1. $\frac{3x-x^3}{1+3x^2}$ 2. $\frac{3x+x^3}{1+3x^2}$

3. $\frac{3x-x^3}{1-3x^2}$ 4. $\frac{3x+x^3}{1-3x^2}$

Solution : (3)

$$\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow \tan^{-1} y = \tan^{-1} x + 2\tan^{-1} x$$

$$\Rightarrow \tan^{-1} y = 3\tan^{-1} x$$

$$\Rightarrow \tan^{-1} y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

$$\Rightarrow y = \left(\frac{3x-x^3}{1-3x^2}\right)$$

54. If the function.

$$\begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases}$$

is differentiable, then the value of k+m is:

1. $\frac{10}{3}$ 2. 4

3. 2 4. $\frac{16}{5}$

Solution: (3)

$$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases}$$

g(x) is differentiable

$$\Rightarrow g'(3^-) = g'(3^+) \Rightarrow \left(\frac{k}{2\sqrt{x+1}}\right)_{x=3} = m$$

$\Rightarrow$ k = 4m ....(1)

Next, g(x) is continuous as it is differentiable.

So, $g(3^-) = g(3^+) \Rightarrow 2k = 3m+2$ ....(2)

From (1) and (2),

we get m = 2/5 $\Rightarrow$ k + m = 2

55. The mean of the data set comprising of 16 observations is 16. if one of the observation valued 16 is deleted and three new observations valued 3,4 and 5 are added to the data, then the mean of the resultant data, is

1. 15.8 2. 14.0
3. 16.8 4. 16.0

Solution:(2)

Sum = S = $16 \times 16 = 256$.

Mean = $\frac{S-16+3+4+5}{18} = \frac{256-16+12}{18} = 14$.

56. The integral

$$\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)}dx$$ is euql to:

1. 1 2. 6
3. 2 4. 4

Solution: (1)

$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)}dx$$

$$= \int_2^4 \frac{\log x^2}{\log x^2 + \log(6-x)^2}dx$$

$$= \int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2}dx$$

$$\Rightarrow 2I = \left[X\right]_2^4 \Rightarrow I = 1.$$

57. Let α and β be the roots of equation

$x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for n $\geq$ 1, then

the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to :

1. 3 2. -3
3. 6 4. -6

Solution: (1)

$x^2 - 6x - 2 = 0 \Rightarrow \alpha + \beta = 6, \alpha\beta = -2$

Now, $\frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$

$$\Rightarrow \frac{(\alpha^{10} - \beta^{10}) + \alpha\beta(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^9(\alpha+\beta) - \beta^9(\alpha+\beta)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{(\alpha^9 - \beta^9)(\alpha+\beta)}{2(\alpha^9 - \beta^9)} = \frac{(\alpha+\beta)}{2} = 3.$$

58. Let f(x) be polynomial of degree four having exterme values at x=1 and x=2.

If $\lim_{x\to 0}\left[1 + \frac{f(x)}{x^2}\right]$=3, then f(2) is equal to :

1. 0 2. 4
3. -8 4. -4

Solution: (1)

Let $f(x) = x^2(ax^2+bx+c)$.

$\lim_{x\to 0}\left[1 + \frac{f(x)}{x^2}\right] = 3 \Rightarrow c + 1 = 3 \Rightarrow c = 2.$

Again, $f(x) = x^2(ax^2+bx+c)$

$\Rightarrow f'(x) = x^2(2ax+b) + 2x(ax^2+bx+c)$

$\Rightarrow f'(x) = 4ax^3 + 3bx^2 + 2cx$

We have, $f'(1) = 0 \Rightarrow 4a + 3b + 4 = 0$ ....(1)

and $f'(2) = 0 \Rightarrow 32a + 12b + 8 = 0$ ....(2)

From (1) and (2) we have,

a = 1/2 and b = -2

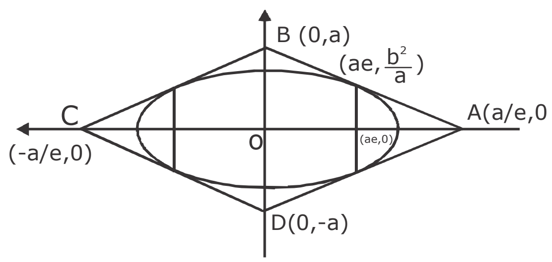
$\therefore$ f(2) = 0.

59. The area (in sq. units) of the quardrilateral formed by the tangents at the end points of the latera recta to the ellipse

$\frac{x^2}{9} + \frac{y^2}{5}$=1 is

1. $\frac{27}{2}$ 2. 27
3. $\frac{27}{4}$ 4. 18

Solution: (2)



Equation of tangent at $\left(ae, \frac{b^2}{a}\right)$ is given by,

$$\frac{aex}{a^2} + \frac{b^2y}{ab^2} = 1 \Rightarrow \frac{ex}{a} + \frac{y}{a} = 1$$

The tangent meets the axes at A(a/e,0) and B(0,a).

Similarly we can find out C(-a/e,0) and

D(0,-a).

Area of quadrilateral = $\frac{1}{2} \times 2a \times \frac{2a}{e} = \frac{2a^2}{e}$

$$= \frac{(2)(9)}{(2/3)} = 27 \begin{cases} \because 9(1 - e^2) = 5 \\ \Rightarrow e^2 = 1 - \frac{5}{9} = \frac{4}{9} \Rightarrow e = 2/3 \end{cases}$$

60. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is:

1. $220\left(\frac{1}{3}\right)^{12}$
2. $22\left(\frac{1}{3}\right)^{11}$
3. $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$
4. $55\left(\frac{2}{3}\right)^{10}$

Solution: Theoretically the question is wrong.