Jee Mains Solution 2016 Test (Code E)

PHYSICS

1. (a)

Mean value= $\frac{90+91+95+92}{4}$ = 92 sec

mean absolute error = $\frac{2+1+3+0}{4}$ = 1.5 sec

Reported main time = 92 $\pm$ 2 sec

2. (b)

The 1 distance will be either $\left(\frac{R}{\sqrt{2}}\right)$ or $\left(\frac{R}{\sqrt{2}}+a\right)$

Therefore $\vec{L}$ = mv $\left(\frac{R}{\sqrt{2}}-a\right)\hat{k}$ will never be possible

3. (c)

Total loss of energy = mgh = 2 mg
we during slide (energy equation)

$mg = \frac{\mu mg\cos\theta}{\sin\theta} \Rightarrow \mu = \frac{1}{2\sqrt{3}} \approx 0.29$

during horizontal motion (energy equation)

$mg = \mu mgx \Rightarrow x = \frac{1}{\mu} = \frac{1}{0.29} \approx 3.5$ m

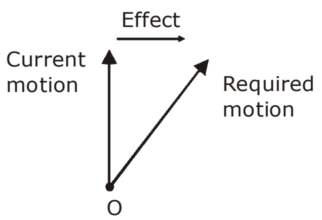
4. (d)

w/d = mgh$\times$1000 = $9.8\times10^{\mu}$ J
Energy / kg = 20% of 3.8 $\times10^{7}$J = 0.76$\times10^{7}$J

Fat burnt = $\frac{9.8\times10^{4}}{0.76\times10^{7}}$ = 12.89$\times10^{-3}$ kg

5. (b)

The roller with try to move along the equidistance line from AB and CD
$\therefore$ it will move towards right



6. (d)

$v_0 = \sqrt{gr}$ $v_e = \sqrt{2gr}$

$\therefore$ increase required = $\sqrt{2gr} - \sqrt{gr} = \sqrt{gr}\left(\sqrt{2}-1\right)$

7. (a)

Using aligation median method

To = $\frac{12\times20+4\times40}{12+4} = \frac{400}{16}$ = 25°C

Now $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$

$$\frac{12}{60 \times 60 \times 24} = \frac{1}{2} \times 15 \times \alpha$$

$$\Rightarrow \alpha = \frac{1}{60 \times 60 + 15} = 1.85 \times 10^{-5}/°C$$

8. (b)

PV^n = constant, this process is polytripic process

$c = C_v \frac{\gamma - n}{1 - n}$; $\gamma = \frac{C_p}{C_v}$; From equation (1)

$c - cn = rC_v - nC_v$

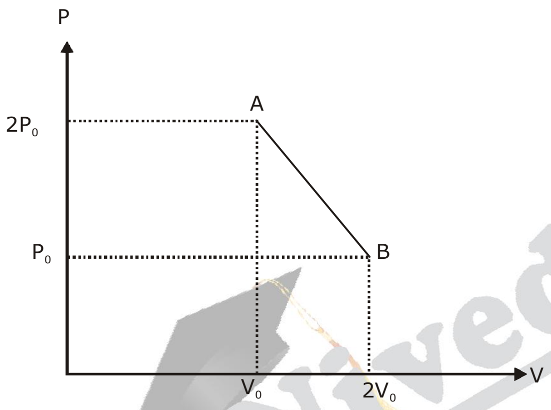
$n (c - C_v) = \gamma C_v$

$n (c - C_v) = c - \gamma C_v$

$n (c - C_v) = c - \frac{C_p}{C_v}$

$\because \gamma = \frac{C_p}{C_v}$; $n = \frac{C - Cp}{C - C_v}$

9. (a)



The graph is a stright line ; p= mv+c, where m is slope.

Point A and B should satisfy the above equation

$2P_0 = mV_0 + C$....... (i); $P_0 = m2V_0 + C$....... (ii)

after solving equation (i) and (ii), we get

$c = 3P_0$ and $m = \frac{-P_0}{V_0}$

Now, P = mv + c; and for ideal gas

pv = nRT; (mv+c) v = nRT;

$mv^2 + cv = nRT$ .... (iii)

for max temp

$2mv + c = nR\frac{dT}{dv}$; when $\frac{dT}{dv} = 0 \Rightarrow$; $v = \frac{-C}{2m}$

after putting the values of c and m, we get $v = \frac{3V_0}{2}$

i.e. T is maximum when $V = \frac{3V_0}{2}$

from (iii)

$$T_{max} = \frac{1}{nR}\left[\frac{-P_0}{V^0} \times \frac{9V_0 2}{4} + 3P_0 \times \frac{3V_0}{2}\right]$$

$$T_{max} = \frac{9PV_0}{4nR}$$

10. (d)

$v = \omega\sqrt{A^2 - x^2}$ .... (i)

at $x = \dfrac{2A}{3}$

$v = \omega\sqrt{A^2 - \dfrac{4A^2}{9}} \Rightarrow v = \dfrac{\omega A}{3}\sqrt{5}$

the, velocity is imreased 3 times

$v_1 = 3v = \omega A\sqrt{5}$

from (i)

$v_1^2 = \omega^2\left(A_1^2 - x^2\right) \Rightarrow$ at $x = \dfrac{2A}{3}$

$\omega^2 A^2 5 = \omega^2\left(A_1^2 - \dfrac{4A^2}{9}\right) \Rightarrow A_1 = \dfrac{7A}{3}$

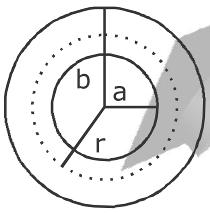
11. (c)

$v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{\mu xg}{x}} = \sqrt{xg}$

$\dfrac{dx}{dt} = \sqrt{xg} \Rightarrow \int \dfrac{dx}{xg} = \int_0^t dt$

$2\sqrt{\dfrac{x}{g}} = t \Rightarrow t = 2\sqrt{\dfrac{20}{10}} = 2\sqrt{2}$ sec

12. (a)



Making a Gaussian surface at a distance from centre

$q = \int_a^r \rho dv = \int_a^r \dfrac{A}{r} 4\pi r^2 dr$

$q = 2\pi(r^2 - a^2)$

total charge enclosed by the gaussian surface

$q_1 = 2\pi\left(r^2 - a^2\right) + Q$

from gauss formula

$E.4\pi r^2 = \dfrac{q_1}{\varepsilon_0}$

$E.4\pi r^2 = \dfrac{2\pi(r^2 - a^2) + Q}{\varepsilon_0}$

$E = \dfrac{2\pi A\left(r^2 - a^2\right) + Q}{4\pi r^2 \varepsilon_0}$

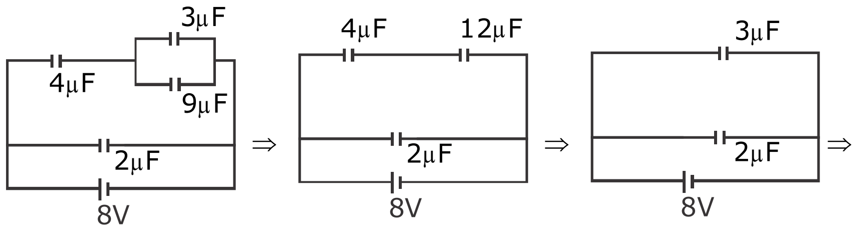
since E is same at a and b

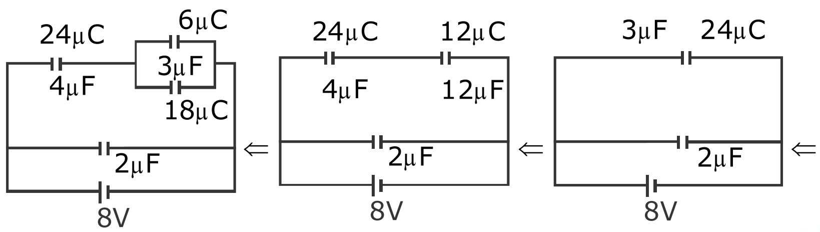
$$\frac{Q}{4\pi\varepsilon_0 a^2} = \frac{2\pi A\left(r^2 - a^2\right) + Q}{4\pi\varepsilon_0 a^2}$$

after solving we get

$$A = \frac{Q}{2\pi a^2}$$

13. (c)





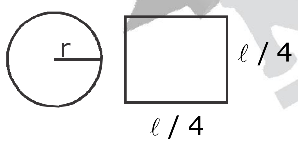
$\therefore$ Total change = (24 + 18) μC = 42 μC

$$E = \frac{1}{4\pi E_0} \cdot \frac{Q}{r^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{(30)^2} = 420\ \mu / C.$$

14. (c)

Cu is metal so at room temperature α will be small and positive.
Si is semiconductor at room temperature α will be high and negative.
$\therefore$ Linear increase for Cu and eaxaponential decrease for Si.

15. (d)



$$\ell = 2\pi r \Rightarrow r = \frac{\ell}{2\pi}$$

$$\Rightarrow r = \frac{\ell}{2\pi}$$

Magnetic field at the centre of circle

$$B_A = \frac{\mu_0 I}{2r}$$

$$B_A = \frac{\mu_0 I \pi}{\ell}$$

Magnetic field at the centre of square

$$B_B = \frac{2\mu_0 I}{\pi a^2}\ \sqrt{2a^2}\ \left[a = \frac{\ell}{4}\right]$$

$$B_B = \frac{8\sqrt{2}\mu_o I}{8\sqrt{2}}$$

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

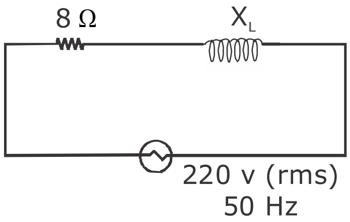
16. (d)

Hysterists loss is the area enclosed by the curve and area is minimum for graph (B)

17. (d)

Resistance of arc lamp

$$R = \frac{V}{I} \Rightarrow R = \frac{80}{10} \Rightarrow R = 8\Omega$$



$$Z = \sqrt{(8)^2 + X_L{}^2}$$

$$Z = \sqrt{(8)^2 + (WL)^2}$$

$$V = I\,Z \Rightarrow 220 = 10\sqrt{64 + (50)^2 L^2} \Rightarrow L = 0.65\ \text{H}$$

18. (a)

For bigger frequency, energy will also be bigger so the increasing order of energy is Radiowave. Yellow light, Blue light, X-ray.

19. (c)

$$m = \frac{h_i}{h_o} \Rightarrow hi = m \times ho = 20 \text{ times taller.}$$

20. (c)

Geometrical spread = a

Diffraction spread = $\left(\frac{\lambda}{2a}\right)L = \frac{\lambda}{2a}L$

Sum (b) = $a + \frac{\lambda L}{2a}$

For b to be minimum

$$\frac{db}{da} = 0 \Rightarrow 1 - \frac{\lambda L}{2a^2} = 0 \Rightarrow a = \sqrt{\frac{\lambda L}{2}}$$

and $b_{min} = \sqrt{\frac{\lambda L}{2}} + \sqrt{\frac{\lambda L}{2}} = \sqrt{2\lambda L}$

No answer is correct.

21. (a)

$$\frac{h_C}{\lambda} = \phi + \frac{1}{2}mv^2$$

now, $\frac{4}{3}\frac{h_C}{\lambda} = \phi + \frac{1}{2}mv'^2$

$$\frac{4}{3}\left[\phi + \frac{1}{2}mv^2\right] = \phi + \frac{1}{2}mv'^2$$

$$\therefore \frac{1}{2}mv^2 = \frac{\phi}{3} + \frac{1}{2}mv^2\ \frac{4}{3}.\frac{1}{2}mv^2$$

$$v'^2 = \frac{2\phi}{3m} + \frac{4v^2}{3} \Rightarrow v' = \sqrt{\frac{2\phi}{3m} + \frac{4}{3}v^2}$$

$$\therefore v' > \sqrt{\frac{4}{3}}v$$

22. (d)

In 80 minutes

A $\rightarrow$ 4 half lives $\rightarrow \frac{15}{16}$ decay

B $\rightarrow$ 2 half lives $\rightarrow \frac{3}{4}$ decay

Required ratio = $\frac{15}{16} : \frac{3}{4} = 5 : 4$

23. (c)

0 + 0 + 0 + 0 = 0
1 + 0 + 0 + 0 = 1
0 + 1 + 0 + 0 = 1
Which is only satisfying by OR gate

24. (a)

In amplitude modulation the amplitude of light frequency carrier wave is made to vary in proportion to the amplitude of the audio signal

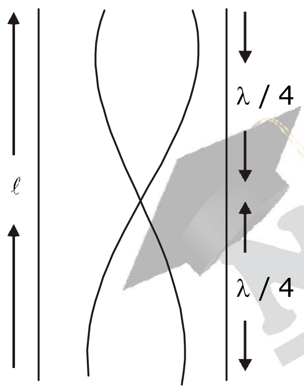
25. (b)

Zero error = – 5 divisions
Reading = 0.5 + 25 divisions
correct reading = 0.5 + 25 div + 5 div = 0.5 + 30 div

1 div = $\frac{0.5}{50}$ = 0.01 mm

$\therefore$ correct reading = (0.5 + 30 × 0.01) = 0.30 mm

26. (d)

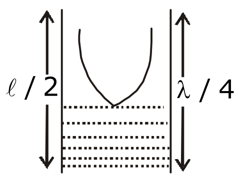


For open pipe

I = $\ell = \frac{\lambda}{4} + \frac{\lambda}{4} \Rightarrow \ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\ell$

f = $\frac{\vartheta}{2\ell}$ ....(i)

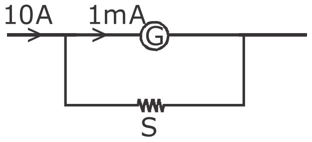
when it is dipped in water



$\frac{\ell}{2} = \frac{\lambda}{4} \Rightarrow \ell = \frac{\lambda}{2}$

which is same, so frequency will also be same

27. (a)



G = 100 Ω
i_g = 1 mA = 1 × 10^{-3} A
S = ?
i = 10A
from formula
$i_g.G = (i - i_g).S$
1 × 10^{-3} × 100 = (10 – 1 × 10^{-3}) S
S = 0.01 – 2.

28. (a)

$A + \delta = i + e \rightarrow A = i + e - \delta = 74°$

Let δ = 40° be minimum deviation.

$$\mu = \frac{\sin\left(\frac{\delta m + A}{2}\right)}{\sin A/2} = \frac{\sin 57°}{\sin 37°} \approx 1.39$$

$\therefore$ μ will surely be less than 1.5.

29. (a)

The graphs are of simple diode, Zener diode, solar diode and light dependent resistance respectively.

30. (b), (d)

We know that $\beta = \frac{\alpha}{1-\alpha}$

So $\frac{1}{\alpha} = \frac{1}{\beta} + 1$ and $\alpha = \frac{\beta}{1+\beta}$ are correct and remaining are incorrect.

CHEMISTRY

31. (b)

It is obvious

$$C_xH_y + \left(\frac{y}{4} + x\right) O_2 \longrightarrow x\,CO_2 + \frac{y}{2} H_2O$$

15x mL $\left(x + \frac{y}{4}\right)$ 15 mL 15x mL

volume of oxygen used = 375 × $\frac{20}{100}$ = 75 mL.

volume of air which is not reacted = 300 mL.
Total volume of gaseous = vol. of CO_2 + vol. of air
which is not reacted = 330 mL.
thus vol. of CO_2 = 30 mL
thus x = 2, y = 12

32. (c)

Let no. of moles of gas in left bulb = n_1
and no. of moles of gas in right bulb = n_2
Since both bulb contain same no. of moles initially therefore $n_1 = n_2$
pv = nRT

$$n = \frac{PV}{RT}$$

$$n_1 = \frac{P_iV}{RT_1}, \; n_2 = \frac{P_iV}{RT_1}$$

Therefore total no. of moles $(n_1 + n_2) = 2\dfrac{P_iV}{RT_1}$

after heating the left bulb

$n_1 = \dfrac{P_fV}{RT_2}$, $n_2 = \dfrac{P_fV}{RT_1}$; $n_1 + n_2 = \dfrac{P_fV}{RT_2} + \dfrac{P_fV}{RT_1}$

or $2\dfrac{P_iV}{RT_1} = \dfrac{P_fV}{R}\left(\dfrac{1}{T_2} + \dfrac{1}{T_1}\right)$

or $2\dfrac{P_i}{T_1} = P_f\left(\dfrac{T_1 + T_2}{T_1T_2}\right)$

or $P_f = 2P_i\left(\dfrac{T_2}{T_1 + T_2}\right)$

33. (d)

$\lambda = \dfrac{h}{mv}$ or $\dfrac{h}{\lambda} = mv$

$\dfrac{1}{2}mv^2 = eV$

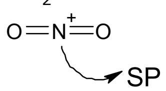
or $\dfrac{1}{2}m^2v^2 = eV$

$(mv)^2 = 2meV$

$mv = \sqrt{2meV}$

34. (a)

NO_2^+



35. (d)

$C + O_2 \rightarrow CO_2 \quad \Delta_CH = -393.5\,kJ\,mol^{-1}$ ...(1)

$CO + \dfrac{1}{2}O_2 \rightarrow CO_2 \quad \Delta_CH = -283.5\,kJ\,mol^{-1}$ ...(2)

(1) — (2)

$C + \dfrac{1}{2}O_2 \rightarrow CO$

$\Delta_fH = \{-393.5 - (-283.5)\}$ kJ mol^{-1}

$= -393.5 + 283.5$

$= -110$ kJ mol^{-1}

36. (c)

$n_{C_6H_{12}O_6} = \dfrac{18}{180} = 0.1$

$n_{H_2O} = \dfrac{178.2}{18} = 9.9$

$p_{H_2O} = X_{H_2O} \times 760$ mm

$= \dfrac{9.9}{0.1 + 9.9} \times 760$mm

$= 752.4$ mm

37. (c)

$A + B \rightleftharpoons C + D$

Initially 1M 1M 1M 1M

At eqb. (1–a) (1–a) (1+a) (1+a)

$$Kc = \frac{[C][D]}{[A][B]}$$

or $100 = \frac{(1+a)(1+a)}{(1-a)(1-a)}$

or $100 = \frac{(1+a)^2}{(1-a)^2}$

or $\sqrt{100} = \frac{1+a}{1-a}$

or $10 = \frac{1+a}{1-a}$

or 10 – 10a = 1 + a

or 11a = 9

$a = \frac{9}{11}$ = 0.818. Thus at eqb. [D] = 1 + a

= 1 + 0.818 = 1.818

38. (d)

39. (b)

For first order reaction

$k = \frac{2-303}{t}$ by $\frac{[A]_0}{[A]_t}$

T = 50 min

$[A]_0$ =0.5 M

$[A]_t$ = 0.125 M

$k = \frac{2.303}{50} \log \frac{0.5}{0.125} = \frac{2.303}{50}\ 2 \log 2$

$= \frac{2.303}{50} \times 2 \times 0.3010$

= 0.02772812

$2H_2O_2 \rightarrow 2H_2O + O_2$

$\frac{\Delta[H_2O_2]}{\Delta T} = k\ [H_2O_2]$

= 0.02772812 × 0.05

= 1 3.64 × 10^{-4}

rate of formation of $O_2 = \frac{\Delta[O_2]}{\Delta t}$

$= \frac{-1}{2}\ \frac{\Delta[H_2O_2]}{\Delta T}$

$= \frac{1}{2} \times 13.64 \times 10^{-4}$

= 6.8 × 10^{-4}

40. (c)

41. (d)

Generally elements of group first has lowest value of ionization energy. Sc has higher value of ionization energy.

42. (c)

Sulphide ores are concentrated by froth floation method so Galena (PbS) is concetrated by this method.

43. (c)

44. (d)

Li reacts with O_2 to from oxide Li_2O, Na reacts with O_2 to from peroxide Na_2O_2 and all other alkali metals reacts with O_2 to from superoxide.

45. (a)

$4Zn + 10\,NHO_3\,(\text{dilute}) \rightarrow 4\,Zn\,(NO_3)_2 + 5H_2O + N_2O$

$Zn + 4HNO_3\,(\text{conc.}) \rightarrow Zn\,(NO_3)_2 + 2\,H_2O + 2\,NO_2$

46. (a)

orthophospherous acid

H_3PO_3

$3 + n - 6 = 0$

$n = +3$

pyrophospherous acid

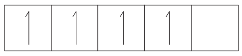
$H_4P_2O_5$

$4 + 2n - 10 = 0$

$n = +3$

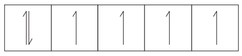
47. (b)

$[Cr(H_2O)_6]^{2+} \Rightarrow Cr^{2+}$, $[Ar]3d^4$



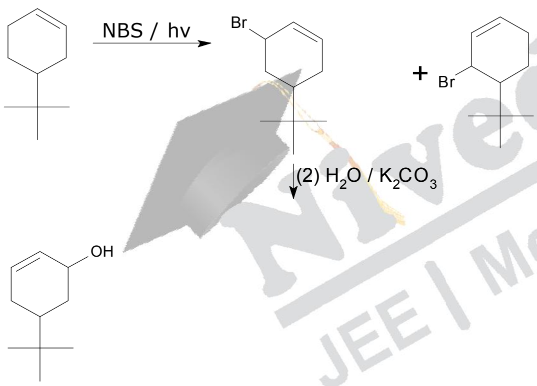
Four unpaired e^-S

$[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+}$, $[Ar]3d^6$

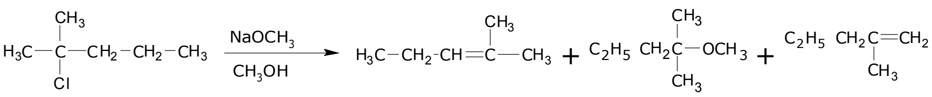


Four unpaired e^-S

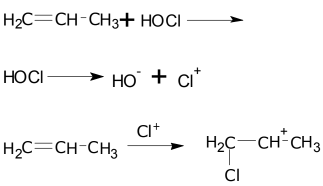
48. (b)
49. (b)
50. (c)
51. (d)
52. (d)



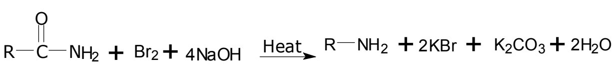
53. (b)
54. (a)



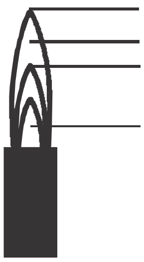
55. (b)



56. (d)



57. (d)
58. (c)
59. (b)
60. (b)



MATHEMATICS

61. (c)

$$f(x)+2f\left(\frac{1}{x}\right)=3x \quad ......(i)$$

$$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x} \quad ....(ii)$$

On solving, $f(x) = \frac{2}{x}-x$

Now, $f(x) = f(-x)$

$\Rightarrow \frac{2}{x}-x = -\frac{2}{x}+x$

$\Rightarrow 2x-\frac{4}{x}=0$

$\Rightarrow x-\frac{2}{x}=0$

$\Rightarrow x^2-2=0$

$\Rightarrow x = \sqrt{2}, -\sqrt{2}$

62. (d)

$Re[(2+3i\sin\theta)(1+2i\sin\theta)]=2-6\sin^2\theta=0$

$\Rightarrow \sin^2\theta=\frac{1}{3}$

$Re[(2+3i\sin\theta)(1+2i\sin\theta)]=2-6\sin^2\theta=0$

63. (b)

$\left(x^2-5x+5\right)^{x^2+4x-60}=1$

Case-I

$x^2 + 4x - 60 = 0$

$x = -10$

$x=6$

Case-II

$x^2 - 5x + 5 = 1$

$x^2 - 5x + 4 = 0$

$x = 1$

$x = 4$

Case-III

$x^2 - 5x + 5 = -1$

$x^2 - 5x + 6 = 0$

$x = 2$ or 3

For $x = 2$

$x^2 + 4x - 60 = -48$

For $x = 3$

$x^2 + 4x - 60 = -39$

$\therefore\ x = 2$

Sum of all real value = 3

64. (b)

$$A = \begin{bmatrix} 5a & b \\ 3 & 2 \end{bmatrix}$$

A adj. A = AA^T

$$\Rightarrow \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix} = \begin{bmatrix} 25a+b & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$$

$$\Rightarrow \left(\begin{matrix} 15a-2b=0 & \Rightarrow b=3,\ a=\frac{1}{5} \\ 10a+3b=13 & 5a+b=10=13 \end{matrix} \right]$$

$$\Rightarrow \left(\begin{matrix} 15a-2b=0 & \Rightarrow b=3,\ a=\frac{2}{5} \\ 10a+3b=13 & \Rightarrow 5a+b=2+3=5 \end{matrix} \right.$$

65. (d)

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - \lambda = 0$

$\Rightarrow \lambda(\lambda+1)(\lambda-1) = 0$

$\Rightarrow \lambda = 0, 1, -1$

66. (d)

Position of word 'SMALL' = 12 + 24 +12 +3 + 6 +1 = 58^{th}

67. (d)

No. of terms = 28

$\Rightarrow \frac{1}{2}(n + 1)(n + 2) = 28$

$\Rightarrow n = 6$

sum of coefficients = $3^6 = 729$

68. (b)

$a_2 = a + d$, $a_5 = a + 4d$, $a_9 = a + 8d$

$(a_5)^2 = a_2 \times a_9$

$(a + 4d)^2 = (a + d)(a + 8d)$

$a^2 + 16d^2 + 8ad = a^2 + 9ad + 8d^2$

$8d^2 = ad$

$a = 8d$

$$\text{ratio} = \frac{a+4d}{a+d} = \frac{8d+4d}{8d+d} = \frac{12d}{9d} = \frac{4}{3}$$

69. (b)

$$S_n = \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2$$

$$S_n = \frac{1}{25}[8^2 + 12^2 + 16^2 + 20^2 + \ldots.]$$

$$S_n = \sum_{n=1}^{10} \frac{1}{25}\left[(4n+4)^2\right]$$

$$= \sum_{n=1}^{10} \frac{16}{25}\left[n+1^2\right]$$

$= \sum_{n=1}^{10} \frac{16}{25}\left[n^2 + 2^n + 1\right]$ 35.11

$= \frac{16}{25}\left[\frac{10.11.21}{6} + \frac{10.11}{2} + 10\right]$

$= \frac{16}{25}\left[385 + 110 + 10\right]$

$= \frac{16}{25} \times 505 = \frac{16}{5} \times 101 \Rightarrow m = 101$

70. (c)

$P = \lim_{x \to \infty} + \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$

$= \lim_{e^{x \to 0}} + \frac{1}{2x} \times \tan^2 \sqrt{x}$

$= e^{\frac{1}{2}}$

$\Rightarrow \log p = \frac{1}{2}$

71. (b)

g(x) = f(f(x)

$\Rightarrow g'(x) = f'(f(x))f'(x)$

$\Rightarrow g'(0) = f'(f(0))\, f'(0)$

For $x \to x, \log 2 > \sin x$

$\therefore f(x) = \log 2 - \sin x \quad \therefore f'(x) = -\cos x \Rightarrow f'(0) = -1$

Also, $x \to \log 2,\ \log 2 > \sin x \quad \therefore f(x) = \log 2 - \sin x$

$\therefore f'(x) = -\cos x \Rightarrow f'(\log 2) = -\cos(\log 2)$

$\therefore g'(0) = (-\cos(\log 2))\,(-1) = \cos(\log 2)$

72. (d)

$f(x) = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$

$= \tan\left(\frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) - \sin(x/2)}\right)$

$= \tan^{-1}\frac{1+\tan(x/2)}{1-\tan(x/2)}$

$= \tan^{-1}\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

$= \frac{\pi}{4} + \frac{x}{2}$

$f'(x) = \frac{1}{2}$

$\Rightarrow$ slope of normal = −2

at $x = \frac{\pi}{6}$, $f\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$

$\therefore$ equation of normal is

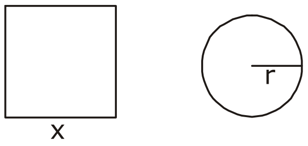
$y - \frac{\pi}{3} = (-2)\left(x - \frac{\pi}{6}\right)$

$\Rightarrow y + 2x = \frac{2\pi}{3}$

73. (c)

$4x + 2\pi r = 2$

$\Rightarrow r = \dfrac{1-2x}{\pi}$



$A = \text{Area} = x^2 + \pi \times \dfrac{(1-2x)^2}{\pi 2} = x^2 + \dfrac{(1-2x)^2}{\pi}$

$\dfrac{dA}{dx} = 2x + \dfrac{2}{\pi}(1-2x)(-2) = 0$

$\Rightarrow x - \dfrac{2(1-2x)}{\pi} = 0$

$\Rightarrow x = \dfrac{2(1-2x)}{\pi} = \dfrac{2}{\pi} - \dfrac{4x}{\pi}$

$\Rightarrow \left(1 + \dfrac{4}{\pi}\right)x = \dfrac{2}{\pi}$

$\Rightarrow (\pi + 4)\pi = 2$

$\Rightarrow x = \dfrac{2}{\pi+4} \Rightarrow \dfrac{x}{2} = \dfrac{1}{\pi+4}$

$\therefore r = \dfrac{1-2x}{\pi} = \dfrac{1}{\pi} - \dfrac{2}{\pi} \times \dfrac{2}{\pi+4} = \dfrac{1}{\pi} - \dfrac{4}{\pi(\pi+4)}$

$= \dfrac{\pi + 4 - 4}{\pi(\pi+4)} = \dfrac{1}{\pi+4}$

So, $\dfrac{x}{2} = r \Rightarrow x = 2r$

74. (b)

$\displaystyle\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3}dx$

$= \displaystyle\int \frac{2x^{12} + 5x^9}{x^{15}(1 + x^{-2} + x^{-5})^3}dx$

$= \displaystyle\int \frac{2x^{-3} + 5x^{-6}}{(1 + x^{-2} + x^{-5})^3}dx$

putting $1 + x^{-2} + x^{-6} = t$,
we get $dt = -(5x^{-6} + 2x^{-3})dx$

$= -\displaystyle\int \frac{dt}{t^3} = -\left(\frac{t^{-2}}{-2}\right) = \frac{1}{2t^2} + c$

$= \dfrac{x^{10}}{2(x^5 + x^3 + 1)^2} + c$

75. (b)

$$\lim_{n\to\infty}\left\{\frac{(n+1)(n+2)......3n}{n^{2n}}\right\}^{1/n}$$

$$= \lim_{\pi\to\infty}\left\{\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n}\right)\left(\frac{n+2}{n}\right).......\right\}^{1/n}$$

$$\lim_{\pi\to\infty}\frac{1}{n}\left[\log\left(1+\frac{1}{n}\right)+\log\left(1+\frac{2}{n}\right)+.....\right]$$

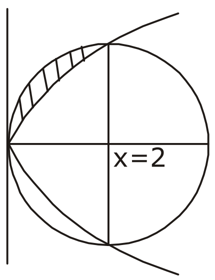
$$= e^{\int_0^2 \log(1+x)dx}$$

$$= e^{3\log 3-2}$$

$$= e^{\log 27} \quad e^{-2}$$

$$= \frac{27}{e^2}$$

76. (b)



$$\text{Area} = \pi\times\frac{(2)^2}{4}-\sqrt{2}\int_0^2\sqrt{x}\ dx$$

$$= \pi-\sqrt{2}\times\frac{2}{3}\left[x^{3/2}\right]_0^2$$

$$= \pi-\frac{2r_2}{3}\times 2\sqrt{2}$$

$$= \pi-\frac{8}{3}$$

77. (d)

$$\left[\frac{ydx-xdy}{d^2}+xdx=0\right.$$

$$\frac{x}{y}+\frac{x^2}{2}=c$$

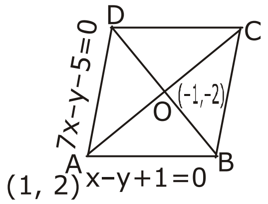
$$x=1,\ y=1,\ -1+\frac{1}{2}=c\Rightarrow c=-\frac{1}{2}$$

$$\text{so, } \frac{x}{y}+\frac{x^2}{2}=-\frac{1}{2}$$

$$x=-\frac{1}{2},\ -\frac{1}{2y}+\frac{1}{8}=-\frac{1}{2}$$

$$\Rightarrow y=\frac{4}{5}\Rightarrow f\left(-\frac{1}{2}\right)=\frac{4}{5}$$

78. (c)



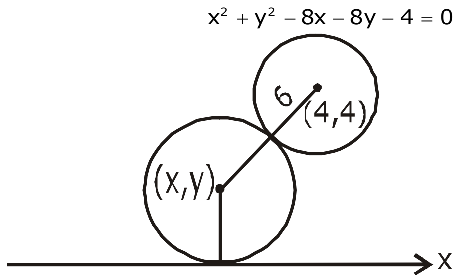
Equation of Diagonal BD is

$y + 2 = -\frac{1}{2}(x + 1)$

$\Rightarrow x + 2y = -5$

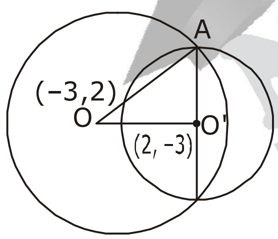
going through options, one vertex is $\left(-\frac{1}{3}, -\frac{8}{3}\right)$

79. (d)



locus of centre of required circle is
$(x - 4)^2 + (x - 4)^2 = (x + 6)^2$
$\Rightarrow x^2 - 8x - 20y - 4 = 0$ which is a parabola

80. (b)



OO′ = $5\sqrt{2}$, O′A =5

$\Rightarrow$ OA = $5\sqrt{3}$

81. (a)
Let P $(2t^2, 4t)$
equation of normal $y + tx = 4t + 2t^3$
Normal must be passes through (0, −6)
$-6 + 0 = 4t + 2t^3$
$\Rightarrow t^3 + 2t + 3 = 0$
$\Rightarrow t = -1$

P = (2, −4), r = $\sqrt{4+4} = \sqrt{8}$
(x−2)2 + (y+4)2
$\Rightarrow x^2 - 4x + 4 + y^2 + 8y + 16 - 8 = 0$
$\Rightarrow x^2 + y^2 - 4x + 8y + 12 = 0$

82. (c)

$2b = C,\ \frac{2b2}{a} = 8$

$\Rightarrow b^2 = 4a$

$a^2 + b^2 = c^2 = 4b^2 = 16a$

$\Rightarrow a^2 + 4a = 16a$

$\Rightarrow a^2 = 12a \Rightarrow a = 12 \Rightarrow c^2 = 16\times12 \Rightarrow c = \sqrt{16\times12}$

$= 8\sqrt{3}$

$\Rightarrow e = \frac{8\sqrt{3}}{12} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$

83. (b)

Equation of line $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

Any point is $(\lambda+1, \lambda-5, \lambda+9)$

It lies on plane

$\Rightarrow (\lambda+1)-(\lambda-5)+(\lambda+9) = 5$

$\Rightarrow \lambda+1-\lambda+5+\lambda+9 = 5$

$\Rightarrow \lambda = -10$

Point is (−9, −15, −1), another is (1, −5, 9)

Distance = $10\sqrt{3}$

84. (d)

$\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$, $\ell x + my - z = 9$

putting (3, −2, −4) , $3\ell - 2m + 4 = 9$

$\Rightarrow 3\ell - 2m = 5$ ..... (i)

Next $2\times\ell + (-1)(m) + 3(1) = 0$

$\Rightarrow 2\ell - m = 3$ ......(ii)

solving (i) and (ii), $\ell = 1$, m=−1

$\ell^2 + m^2 = 2$

85. (d)

$\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$, $\vec{b} \neq \vec{c}$

$\Rightarrow (\vec{a}.\ \vec{c})\ \vec{b} - (\vec{a}.\ \vec{b}) = \frac{\sqrt{3}}{2}\left(\vec{b}+\vec{c}\right)$

$\Rightarrow \left(\vec{a}.\ \vec{c} - \frac{\sqrt{3}}{2}\right)\vec{b} - \left(\vec{a}.\ \vec{b} + \frac{\sqrt{3}}{2}\right)\vec{c} = 0$

$\Rightarrow \vec{a}.\ \vec{b} + \frac{\sqrt{3}}{2} = 0$

$\Rightarrow \vec{a}.\ \vec{b}. = -\frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$, $\Rightarrow \theta = \frac{5\pi}{6}$

86. (b)

$$\text{Variance} = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{4+9+a^2+121}{4} - \left(4+\frac{a}{4}\right)^2$$

$$= \frac{4(134+a^2) - 256 - a^2 - 32a}{16}$$

$$3a^2 - 32a + 280 = 16\left(\frac{7}{2}\right)^2 = 4 \times 49$$

$3a^2 - 32a + 84 = 0$

87. (d)

$$P(E_1) = \frac{1}{6},\ P(E_2) = \frac{1}{6},\ P(E_3) = \frac{1}{2}$$

$$P(E_1 \cap E_2) = \frac{1}{36},\ P(E_2 \cap E_3),\ P(E_2 \cap E_3) = \frac{1}{12},\ P(E_1 \cap E_2) = \frac{1}{12}$$

$P(E_1 \cap E_2 \cap E_3) = 0$

Clearly, E_1, E_2, E_3 are not independent.

88. (c)

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$\Rightarrow (\cos 4x + \cos x) + (\cos 3x + \cos 2x) = 0$

$$\Rightarrow 2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\,.\,\cos\frac{x}{2} = 0$$

$$\Rightarrow 2\cos\frac{5x}{2}\,.\left(\cos\frac{3x}{2} + \cos\frac{x}{2}\right) = 0$$

$$\Rightarrow 2\cos\frac{5x}{2} \times 2\cos x.\ \cos\frac{x}{2} = 0$$

$$\Rightarrow \cos\frac{5x}{2} = 0,\ \cos x = 0,\ \cos\frac{x}{2} = 0$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos\frac{x}{2} = 0 \Rightarrow x = \pi$$

$$\cos\frac{5\pi}{2} = 0 \Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$$

89. (d)

Let speed = S m/sec

Distance = 600S

$$\tan 60° = \frac{y}{x}$$

$$\sqrt{3} = \frac{y}{x}$$

$$y = \sqrt{3}x$$

$$\tan 30° = \frac{y}{x + 600S}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x + 600S}$$

$x + 600S = 3x$

$600S = 2x$
$x = 300S$
Distance = 300S
Time = ?
Speed = S

Time = $\frac{300S}{S} = 300\text{sec} = 5\text{ min}.$

90. (c)

$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$= [p \vee q) \wedge (\sim q \vee q)] \vee (\sim p \wedge q)$

$= [p \vee q) \wedge t] \vee (\sim p \wedge q)$

$= [p \vee q) \vee t] \vee (\sim p \wedge q)$

$= [(p \vee q \vee \sim p) \wedge (p \vee p \vee q)$

$= (t \vee q) \wedge (p \vee q) = t \wedge (p \vee q) = p \vee q$