CHEMISTRY

1. **(1)**

$CH_3COOK + H_2O \rightarrow CH_3COOH + KOH$ Basic

$FeCl_3$ – Acidic solution

$Al(CN)_3$ – Salt of weak acid and weak base

$Pb(CH_3COO)_2$ – Salt of weak acid and weak base

CH_3COOK is salt of weak acid and strong base.

Hence solution of CH_3COOK is basic.

2. **(1)**

Kjeldahl method is not applicable for compounds containing nitrogen in nitro, and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions. Hence only aniline can be used for estimation of nitrogen by Kjeldahl's method.

3. **(3)**

BCl_3 – electron deficient, incomplete octet

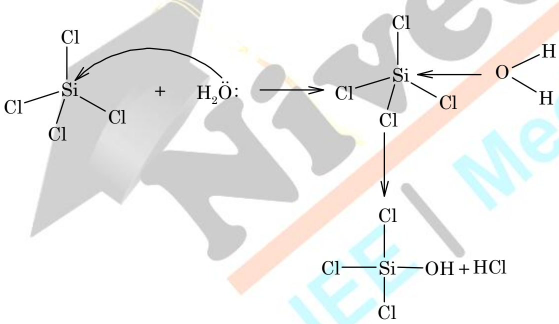
$AlCl_3$ – electron deficient, incomplete octet

Ans-(1) BCl_3 and $AlCl_3$

$SiCl_4$ can accept lone pair of electron in d-orbital of silicon hence it can act as Lewis acid.

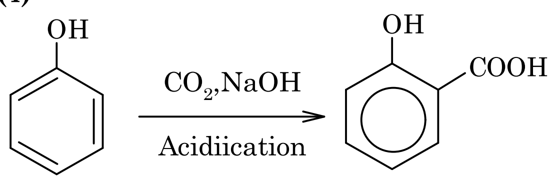
* Although the most suitable answer is (1). However, both option (1) & (3) can be considered as correct answers.

e.g. hydrolysis of $SiCl_4$

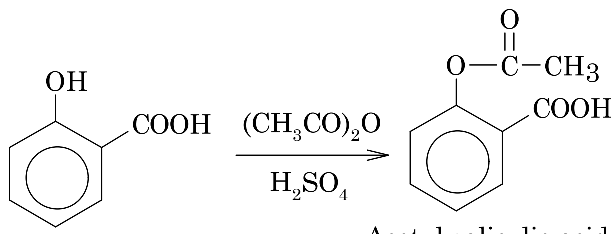


Hence option (3), $AlCl_3$ and $SiCl_4$ is also correct.

4. **(4)**



(Major)



Acetyl salicylic acid
Aspirin

5. **(2)**

⟵ 3.9 — 4.5 ⟶
Pinkish red   Yellow

Weak base is having pH greater than 7. When methyl orange is added to weak base solution, the solution becomes yellow. This solution is titrated by strong acid and at the end point pH will be less than 3.1. Therefore solution becomes pinkish red.

6. **(1)**

In presence of external H^+

$$H_2S \rightleftharpoons 2H^+ + S^{2-},\ K_{a_1}.K_{a_2} = K_{eq}$$

$$\therefore \frac{[H^+]^2[S^{2-}]}{[H_2S]} = 1\times10^{-7} \times 1.2 \times 10^{-13}$$

$$\frac{[0.2]^2[S^{2-}]}{[0.1]} = 1.2\times10^{-20}$$

$$[S^{2-}] = 3 \times 10^{-20}$$

7. **(3)**

$$C_6H_6(l) + \frac{15}{2} O_2(g) \longrightarrow 6CO_2(g) + 3H2O(l)$$

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 + \left(-\frac{3}{2}\right) \times 8.314 \times 298 \times 10^{-3}$$

$$= -3263.9 + (-3.71)$$

$$= -3267.6 \text{ kJ mol}^{-1}$$

8. **(3)**

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4H_2O + Cr_2O_3$$

$$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$$

$$(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$$

$$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$$

Among all the given compounds, only $(NH_4)_2SO_4$ do not form dinitrogen on heating, it produces ammonia gas.

9. (2)

$B_2H_6 + 3O_2 \rightarrow B_2O_3 + 3H_2O$

27.66 of B_2H_6 = 1 mole of B_2H_6 which requires three moles of oxygen (O_2) for complete burning

$6H_2O \rightarrow 6H_2 + 3O_2$ (On electrolysis)

Number of faradays = 12 = Amount of charge

$12 \times 96500 = i \times t$

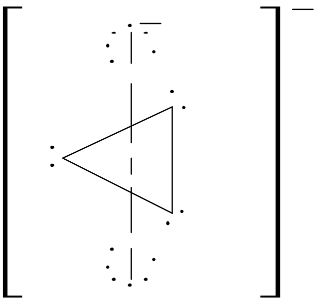
$12 \times 96500 = 100 \times t$

$t = \dfrac{12 \times 96500}{100}$ second

$t = \dfrac{12 \times 96500}{100 \times 3600}$ hour $\Rightarrow t = 3.2$ hours

10. (2)

Structure of I_3^-



Number of lone pairs in $I_3^{\ominus}$ is 9.

11. (2)

$$Al^{3+} \xrightarrow{NaOH} \underset{\text{White gelatinous ppt.}}{Al(OH)_3 \downarrow} \xrightarrow{\text{Excess NaOH}} \underset{\text{Sodium meta aluminate (soluble)}}{NaAlO_2}$$

$$2Al(OH)_3 \xrightarrow{\text{Strong heating}} Al_3O_3 + 3H_2O$$

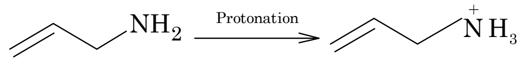
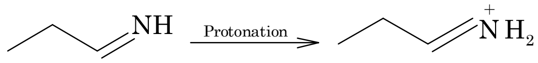
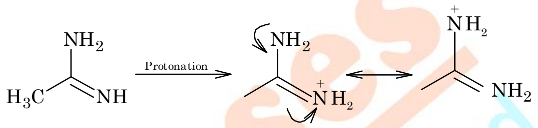
Al_2O_3 is used in column chromatography.

12. (3)

	Electronic configuration	Bond order
He_2^{2-}	$\sigma_{1s}^2 \sigma_{1s}^{*1}$	$\dfrac{2-1}{2} = 0.5$
H_2^-	$\sigma_{1s}^2 \sigma_{1s}^{*1}$	$\dfrac{2-1}{2} = 0.5$
He_2^{2-}	$\sigma_{1s}^2 \sigma_{1s}^{*2}$	$\dfrac{2-2}{2} = 0$
He_2^{2+}	σ_{1s}^2	$\dfrac{2-0}{2} = 1$

Molecular having zero bond order will not be a variable molecule.

13. (2)

(a)	 1° & sp^3
(b)	 sp^2
(c)	 [Equivalent resonance]
(d)	 2° & sp^3

$\therefore$ Correct order of basically : b < a < d < c.

14. (2)

In Frenkel defect, cation is dislocated from its normal lattice site to an interstitial site.

15. (2)

KCl – Ionic bond between K^+ and Cl^-

PH_3 – Covalent bond between P and H

O_2 – Covalent bond between O atoms

B_2H_6 – Covalent bond between B and H atoms

H_2SO_4 – Covalent bond between S and O and also between O and H.

$\therefore$ Compound having no covalent bonds is KCl only.

16. (2)

$[Cr(H_2O)_6Cl_3 \Rightarrow x + 0 \times 6 - 1 \times 3 = 0$

$\therefore x = +3$

$[Cr(C_6H_6)_2 \Rightarrow x + 2 \times 0 = 0$

$x = 0$

$K_2[Cr(CN)_2(O_2)(O_2)NH_3]$

$\Rightarrow 1 \times 2 + x - 1 \times 2 - 2 \times 2 - 2 \times 1 = 0$

$\Rightarrow x - 6 = 0$

$x = +6$

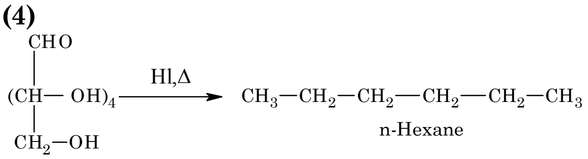
17. (2)

$[Fe(CN)_6]^{4-} + \frac{1}{2} H_2O_2 + H^+ \rightarrow [Fe(CN)_6]^{3-} + H_2O$

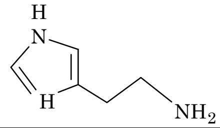
$[Fe(CN)_6]^{3-} + \frac{1}{2} H_2O_2 + OH^{\ominus} \rightarrow$

$[Fe(CN_6)^{4-}] + H_2O + \frac{1}{2} O_2$

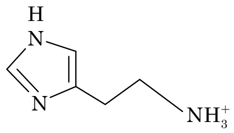
18. (4)



19. (3)

Histamine	

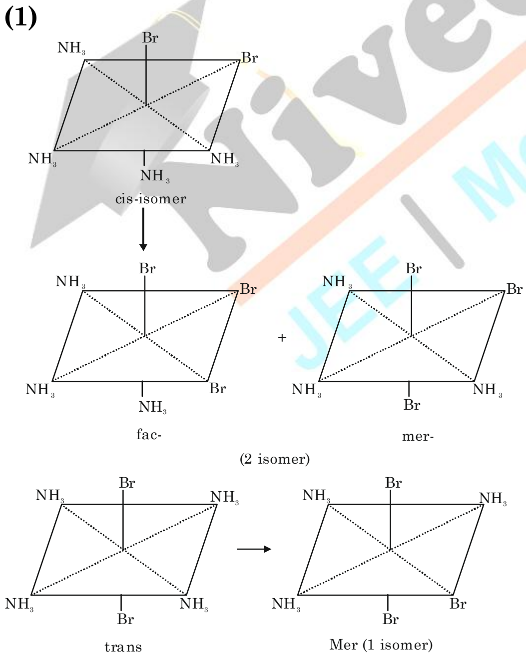
At pH (7.4) major form of histamine is protonated at primary amine.



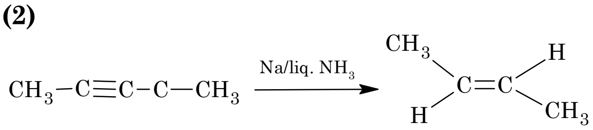
20. (2)

F^- ions make the teeth enamel harder by converting $\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]}$ and $\underset{\text{Fluorapatite}}{[3Ca_3(PO_4)_2.CaF_2]}$

21. (1)



22. (2)



So, option (2) is correct.

23. (3)

Element	Relative mass	Relative mole	Simplest whole number ratio
C	6	$\frac{6}{12} = 0.5$	1
H	1	$\frac{1}{1} = 1$	2

So, X = 1, Y = 2

Equation for combustion of C_XH_Y

$C_XH_Y + \left(X + \frac{Y}{4}\right) O_2 \longrightarrow XCO_2 + \frac{Y}{2} H_2O$

Oxygen atoms required $= 2\left(X + \frac{Y}{4}\right)$

As per information,

$2\left(X + \frac{Y}{4}\right) = 2Z$

$\Rightarrow \left(1 + \frac{2}{4}\right) = Z$

$\Rightarrow Z = 1.5$

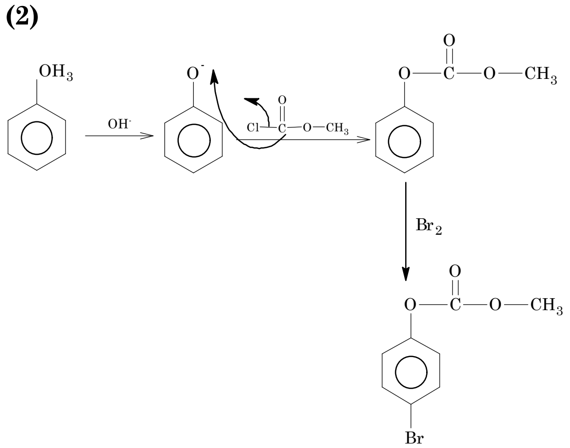
Molecule can be written

$C_XH_YO_Z$

$C_1H_2O_{3/2}$

$\Rightarrow C_2H_4O_3$

24. (2)

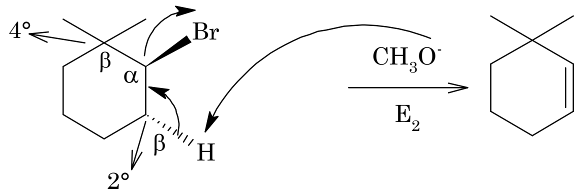


25. (1)

CH_3O^- is a strong base and strong nuclephile, so favourable condition is $S_N2/E2$.

Given alkyl halide is 2° and β C's are 4° and 2°, so sufficiently hindered, therefore, E2 dominates over S_N2.

Also polarity of CH_3OH (solvent) is not as high as H_2O, so E1 is also dominated byE2



26. (4)

Equilibrium constant $K = \left(\frac{A_t}{A_b}\right)e^{-\frac{\Delta H^\circ}{RT}}$

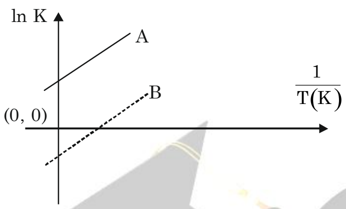
$\ln K = \ln\left(\frac{A_t}{A_b}\right) - \frac{\Delta H^\circ}{R}\left(\frac{1}{T}\right)$

$y = c + mx$

Comparing with equation of straight line,

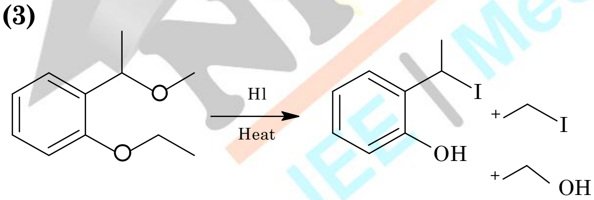
$\text{Slope} = -\frac{\Delta H^\circ}{R}$

Since, reaction is exothermic, ΔH° = –ve, therefore, slope = +ve.



Hence, option (4) is correct.

27. (3)



Hence, option (3) is correct.

28. (2)

Final concentration of $[SO_4^{--}] = \frac{[50 \times 1]}{500} = 0.1$ M

K_{sp} of $BaSO_4$,

$[Ba^{2+}][SO_4^{2-}] = 1 \times 10^{-10}$

$[Ba^{2+}][0.1] = \frac{10^{-10}}{0.1} = 10^{-9}$ M

Concentration of Ba^{2+} in final solution = 10^{-9} M

Concentration of Ba^{2+} in the original solution.

$M_1V_1 = M_2V_2$

$M_1 (500 - 50) = 10^{-9} (500)$

$M1 = 1.11 \times 10^{-9}$ M

So, option (4) is correct.

29. (4)

Assume the order of reaction with respect to acetaldehyde is x.

Condition-1:

Rate = $k[CH_3CHO]^x$

$1 = k[363 \times 0.95]^x$

$1 = k[344.85]^x$ .....(i)

Condition-2:

$0.5 = k[363 \times 0.67]^x$

$0.5 = k[243.21]^x$ .....(ii)

Divide equation (i) by (ii),

$\frac{1}{0.5} = \left(\frac{344.85}{243.21}\right)^x \Rightarrow 2 = (1.414)^x$

$\Rightarrow x = 2$

30. (3)

The solution which shows maximum freezing point must have minimum number of solute particles.

(1) $[Co(H_2O)_3Cl_3] \cdot 3H_2O \rightarrow [Co(H_2O)_3Cl_3]$, i = 1

(2) $[Co(H_2O)_6]Cl_3 \rightarrow [Co(H_2O)_6]^{3+} + 3Cl^-$, i = 4

(3) $[Co(H_2O)_5Cl]Cl_2 \cdot 2H_2O \rightarrow [Co(H_2O)_5Cl]^{2+} + 2Cl^-$, i = 3

(4) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O \rightarrow [Co(H_2O)_4Cl_2]^{2+} + Cl^-$, i = 2

So, solution of 1 molal $[Co(H_2O_3)Cl_3] \cdot 3H_2O$ will have will have minimum number of particles in aqueous state.

Hence, option (3) is correct.

MATHEMATICS

31. (1)

$$\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + 1 + \cos^5 x\right)^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{\cos^{10} x\left(\tan^5 x + \tan^2 x + \tan^3 x + 1\right)} dx$$

$$= \int \frac{\frac{\sin^2 x}{\cos^2 x} \times \sec^6 x}{(\tan^3 x + 1)^2 (\tan^2 x + 1)^2} dx$$

$$= \int \frac{\tan^2 x - \sec^6 x}{(\tan^3 x + 1)^2 \cdot \sec^4 x} dx$$

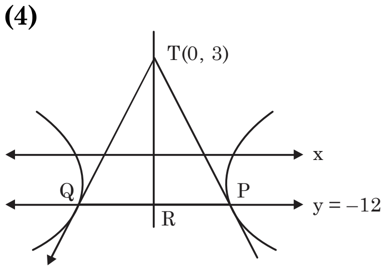
$$= \int \frac{\tan^2 x - \sec^2 x}{(1 + \tan^3 x)^2} dx$$

Putting $1 + \tan^3 x = t \Rightarrow dt = 3\tan^2 x \sec^2 x\, dx$

$$= \frac{1}{3}\int \frac{1}{t^2} dt = -\frac{1}{3t} + c$$

$$= -\frac{1}{3(1 + \tan^3 x)} + c$$

32. (4)

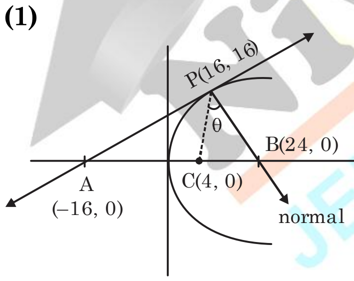


$\text{Area} = \frac{1}{2} \times PA \times TR$ $\begin{cases} TR = 15 \\ PQ = 6\sqrt{5} \end{cases}$

$\frac{1}{2} \times 15 \times 6\sqrt{5}$

$= 45\sqrt{5}$

33. (1)



equation of tangent is
$x - 2y + 16 = 0$

$m_{PC} = \frac{4}{3}$

$m_{PB} = -2.$

$$\therefore \tan\theta = \left| \frac{\frac{4}{3} + 2}{1 + \frac{4}{3} \times (-2)} \right| = 2$$

34. (4)

Let $\vec{u} = x\vec{a} + y\vec{b}$

Now $\vec{u} = \vec{a} \cdot \vec{b} = 0 \Rightarrow 14x + 2y = 0 \Rightarrow y = -7x$ ....(i)

$(\because |\vec{a}|^2 = 14, |\vec{b}|^2 = 2)$

$\vec{u} \cdot \vec{b} = 24 \Rightarrow 2x + 2y = 24$ $(\because \vec{a} \cdot \vec{b} = 2)$

$\Rightarrow x + y = 12$ ......(ii)

From (i) & (ii) $x = -2, y = 14$

$\therefore \vec{u} = -2(2\hat{i} + 3\hat{j} - \hat{k}) + 14(\hat{j} + \hat{k}) = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow |\vec{u}|^2 = 336$

35. (2)

$\alpha = -\omega,\ \beta = -\omega^2$

$\therefore \alpha^{101} + \beta^{107} = -(\omega^2 + \omega) = 1$

36. (4)

$18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow x = \frac{9\pi \pm 3\pi}{36} \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}$

i.e. $\alpha = \frac{\pi}{6},\ \beta = \frac{\pi}{3}$

$\text{Area} = \int_{\pi/6}^{\pi/3} gof(x)\, dx$

$= \int_{\pi/6}^{\pi/3} \cos x\, dx = [\sin x]_{\pi/6}^{\pi/3} = \frac{1}{2}(\sqrt{3} - 1)$

37. (3)

Let $\sqrt{x^2 - 1} = a$

We have, $(x + a)^5 + (x - a)^5$

$= 2\left[{}^5C_0 x^5 + {}^5C_2 x^3 a^2 + {}^5C_4 x^5 a^4\right]$

$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$

$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

Considering odd degree terms,

$2[x^5 + 5x^7 - 10x^3 + 5x]$

Sum of coefficients = 2

38. (2)

$\sum_{k=0}^{12} a_{4k+1} = 416$

$\Rightarrow \frac{13}{2}(2a_1 + 48d) = 416$

$\Rightarrow a_1 + 24d = 31$ ...(i)

$a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$ ...(ii)

from (i) and (ii), $d = 1, a_1 = 8$

Now, $140\,m = \sum_{r=1}^{17} ar^2$

$= \sum_{r=1}^{17}\left[8-1(r-1)\cdot 1\right]^2$

$= \sum_{r=1}^{17}(r+7)^2$

$= \sum_{r=1}^{24} r^2 - \sum_{r=1}^{7} r^2$

$= \dfrac{24\times25\times49}{6} - \dfrac{7\times8\times15}{6}$

$\Rightarrow \quad m = 34$

39. (2)

$\sum_{i=0}^{9}(x_i - 5) = 9 \;\Rightarrow\; \sum_{i=1}^{9} x_i = 54$

Again $\sum_{i=1}^{9}(x_i = 5)^2 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 10\sum_{i=1}^{9} x_i + 225 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 360$

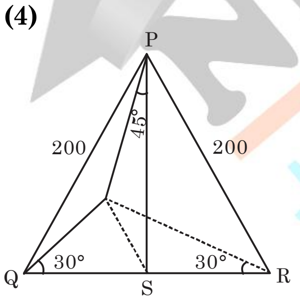
Variance $= \sum_{i=1}^{9} x_i^2 - \left(\dfrac{\sum_{i=1}^{9} x_i}{9}\right)^2$

$= \dfrac{360}{9} - \left(\dfrac{54}{9}\right)^2$

$= 4$

$\Rightarrow$ S.D = 2

40. (4)



Let height of tower

ST = h

In Δ QST, $\tan 30° = \dfrac{ST}{QS}$

$\Rightarrow$ QS $= \sqrt{3}h$ = SR

In Δ STP, ST = PS

In Δ PSQ, PS $= \sqrt{(200)^2 - (\sqrt{3}\,h)^2}$

So, $\sqrt{(200)^2 - 3h^2} = h \Rightarrow h = 100m$

41. (1)

Set A contains all parts inside

$|x| < 1$ and $|y| < 1$

Set B contains all parts inside the ellipse all parts inside that ellipse $\dfrac{(x-1)^2}{9} + \dfrac{y^2}{4} = 1$

clearly ACB.

42. (4)

6 3

no. ways = 6 × 6 × 41

4 1

= 1080

43. (3)

$$h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \frac{\left(x - \frac{1}{x}\right)^2 + 2}{x - \frac{1}{x}}$$

$$= \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}$$

We know, $\dfrac{\left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}}{2} \geq \sqrt{\left(x - \frac{1}{x}\right) \times \frac{2}{\left(x - \frac{1}{x}\right)}}$

$\Rightarrow \left(x - \dfrac{1}{x}\right) + \dfrac{2}{x - \frac{1}{x}} \geq 2\sqrt{2}$

$\Rightarrow$ minimum value of h(x) $= 2\sqrt{2}$

44. (2)

Let $f(x) = x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots.. + \left[\frac{15}{x}\right]\right)$

$= x\left(\frac{1}{x} + \frac{2}{x} + \ldots.. + \frac{15}{x} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots.. + \left\{\frac{15}{x}\right\}\right)\right)$

$= x\left(\frac{15\times16}{2x} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots.. + \left\{\frac{15}{x}\right\}\right)\right)$

$= 120 - x\left\{\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots.. + \left\{\frac{15}{x}\right\}\right\}$

We know $0 \leq \left\{\dfrac{v}{x}\right\} < 1$

$$\Rightarrow\ 0 \le x\left\{\frac{v}{x}\right\} < x$$

$$\Rightarrow\ \lim_{x\to 0^+} x\left\{\frac{v}{x}\right\} = 0$$

$$\Rightarrow\ \lim_{x\to 0} f(x) = 120$$

45. (3)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}dx$$

$$\Rightarrow\ I = \int_{-\pi/2}^{\pi/2} \frac{2^x \sin^2 x}{1+2^x}dx$$

$$\Rightarrow\ 2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\,dx\ = 2\int_0^{\pi/2} \sin^2 x dx$$

$$\Rightarrow\ I = \int_0^{\pi/2} \sin^2 dx = \int_0^{\pi/2} \cos^2 x\,dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$$

46. (1)

Required probability

$$P(R_1)\ P\left(\frac{R_2}{R_1}\right) + P(B_1) \times P\left(\frac{R_2}{B_1}\right)$$

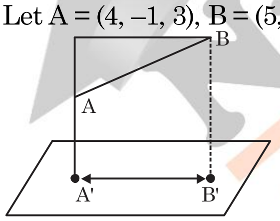
$$= \frac{4}{10}\times\frac{6}{12} + \frac{6}{10}\times\frac{4}{12}$$

R_1, R_2 are drawing red balls in Ist and 2nd draw
B_1 = drawing black ball in Ist draw.

$$= \frac{2}{5}$$

47. (3)

Let A = (4, –1, 3), B = (5, –1, 4)



$$AC = \overrightarrow{AB}\cdot\widehat{AC} = \left(\hat{i}+\hat{k}\right)\cdot\left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$$

$$\text{Now, } A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow\ \text{Length of projection} = \sqrt{\frac{2}{3}}$$

48. (1)

$$8\ \cos x\left\{\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right) - \frac{1}{2}\right\} = 1$$

$$\Rightarrow\ \cos x\left\{\cos^2 x - \sin^2\frac{\pi}{6} - \frac{1}{2}\right\} = \frac{1}{8}$$

$$\Rightarrow\ \cos x\left(\cos^2 x - \frac{3}{4}\right) = \frac{1}{8}$$

$$\Rightarrow\ \frac{4\cos^3 x - 3\cos x}{4} = \frac{1}{8}$$

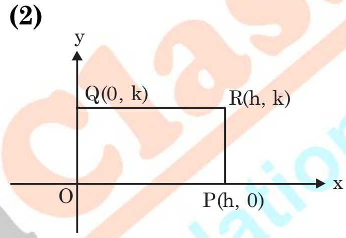
$$\Rightarrow\ \cos 3x = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\Rightarrow 3x = 2n\pi \pm \frac{\pi}{3} \Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$$

In [0, π] sum of solutions $= \frac{\pi}{9} + \frac{2\pi}{3} + \frac{\pi}{9} + \frac{2\pi}{3}$

$$K = \frac{13}{9} \qquad = \frac{3\pi}{9}$$

49. (2)



Equation in PQ is

$$\frac{x}{h} = \frac{y}{k} = 1$$

Putting (2, 3), we get $\frac{2}{h} + \frac{3}{k} = 1$

$\therefore$ Locus will be,

$$\frac{2}{x} + \frac{3}{y} = 1$$

$$\Rightarrow 3x + 2y = xy$$

50. (1)

$$B - 2A = \sum_{r=1}^{40} t_r - 2\sum_{r=1}^{20} t_r$$

$(21^2 + 2.22^2 + ... + 40^2) - (1^2 + 2.2^2 + ... + 20^2)$

= 20[(22 + 24 + ...+ 60) + (24 + 28 + ...+60)]

$$= 20\left[\frac{20}{2}(22+60) + \frac{10}{2}(24+60)\right]$$

$$100\lambda = 100 \times 248\ \Rightarrow\ \lambda = 248$$

51. (3)

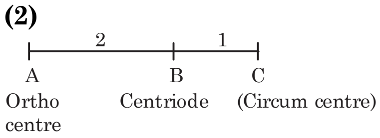
from $y^2 = 6x$, $\frac{dy}{dx} = \frac{3}{y}$

from $9x^2 + by^2 = 16$, $\frac{dy}{dx} = \frac{-ax}{by}$

ATQ, Curves cut at right angle,

$$\frac{-ax}{by}\times\frac{3}{y}=-1 \Rightarrow by^2=27x \Rightarrow b=\frac{27x}{y^2}=\frac{27x}{6x}=\frac{9}{2}$$

52. (2)



$$AC=\frac{3}{2}AB=\frac{3}{2}\times 2\sqrt{10}=3\sqrt{10}$$

Radius of circle with AC as diameter

$$=\frac{3}{2}\sqrt{10}\quad=3\sqrt{\frac{5}{2}}$$

53. (4)

$f(x)=|x-\pi|\left(e^{|x|}-1\right)\sin|x|$

at $x=0$

L.H.D = 0, R.H.d = 0

$\Rightarrow$ f(x) is differentiable.

$\therefore\ S=\phi$

54. (2)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}=(5x-4)(x+4)^2$$

so, eqating, A = –4, B = 5

55. (4)

$\sim(p\vee g)\vee(\sim p\wedge q)$

$=(\sim p\wedge\sim q)\vee(\sim p\wedge q)$

$=\sim p\wedge(\sim q\vee q)$

$=\sim p\wedge t=\sim p$

56. (1)

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix}=0 \Rightarrow k=11$$

$x+11y+3z=0$ ______ (i)

$3x+11y-2z=0$ ______ (ii)

$2x+4y-3z=0$ ______ (iii)

+ (ii) $\Rightarrow x=-5y$

putting it in (i), we get

$-5y+11y+3z=0$

$\Rightarrow z=-2y$

so, $\frac{xz}{y^2}=\frac{(-5y)(-2y)}{y^2}=10$

57. (2)

Case – 1 :

$2\left(3-\sqrt{x}\right)+x-6\sqrt{x}+6=0$

$\Rightarrow\ x-8\sqrt{x}+12=0 \Rightarrow \sqrt{x}=4, 2$

$\Rightarrow x=16$ or $x=4$ (Rejected)

Case – 2 :

Let $x\in[9,\infty)$

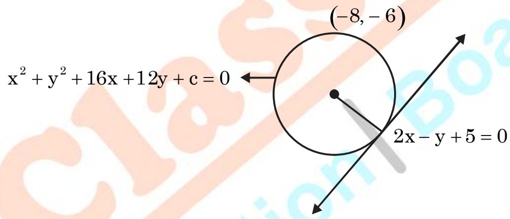
$2\left(\sqrt{x}-3\right)+x-6\sqrt{x}+6=0$

$\Rightarrow\ x-4\sqrt{x}=0 \Rightarrow x=16$ or $x=0$ (Rejected)

So, x = 4, 16 are two solutions.

58. (3)

Tangent at (1, 7) to $x^2=y-6$ is $2x-y+5=0$



Now, $\left|\frac{-16+6+5}{\sqrt{5}}\right|=\sqrt{64+36-C}$

$\Rightarrow C=95$

59. (2)

$\sin x\,dy+y\cos x\,dx=4x\,dx$

$\Rightarrow\ d\,(y\sin x)=4x\,dx$

Integrating, $y\sin x=2x^2+c$

Curve passes through $\left(\frac{\pi}{2},0\right)$

$\Rightarrow\ 0=\frac{\pi^2}{2}+c \Rightarrow c=-\frac{\pi^2}{2}$

Now, $y\sin x=2x^2-\frac{\pi^2}{2}$

$\therefore\ y\left(\frac{\pi}{6}\right)=-\frac{8}{9}\pi^2$

60. (1)

Plane passing through $2x-2y+3z-2=0$ any $x-y+z+1=0$ is given by,

$(2x-2y+3z-2)+\lambda(x-y+z+1)=0$

$\Rightarrow\ (\lambda+2)x-(\lambda+2)y+(\lambda+3)z+(\lambda-2)=0$ ...(1)

Plane (1) and $x+2y-z-3=0$ and $3x-y+2z-1=0$ has infinitely many solution.

$$\Rightarrow \begin{vmatrix} \lambda+2 & -(\lambda+2) & \lambda+3 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}=0$$

$\Rightarrow \lambda = 5$

Equation of required plane is

$$7x - 7y + 8z + 3 = 0$$

Distance from origin $= \dfrac{3}{\sqrt{49+49+64}}$

$$= \frac{3}{9\sqrt{2}} = \frac{1}{3\sqrt{2}}$$

PHYSICS

61. (4)

$2\theta = 60°$

$\theta = 30°$

$d \sin\theta = \lambda$, (d=width of single slit=1 μ m = 10^{-6} m.)

$\lambda = 10^{-6} \sin 30°$

$\lambda = 0.5 \times 10^{-6}$

for Double Slit

$\beta = \dfrac{D\lambda}{d}$ (d = slit separation, D= 50 cm = 0.5 m)

$10^{-2} = \dfrac{0.5 \times 0.5 \times 10^{-6}}{d}$

$d = 25 \times 10^{-6}$ m.

$d = 25\ \mu$ m

62. (4)

K.E. of $\bar{e}$ in n^{th} orbit $= \dfrac{13.6}{n^2} = \dfrac{1}{2}\dfrac{p^2}{m_e}$

$\Rightarrow P = \dfrac{\sqrt{27.2me}}{n} = \dfrac{k_1}{n}$ {Let say $K_1 = \sqrt{27.2me}$

Now $\lambda_n = \dfrac{h}{p} = \dfrac{hn}{k_1} \Rightarrow \lambda_n = k_2 n$

$\dfrac{h}{\lambda n} = 13.6\left[1 - \dfrac{1}{n^2}\right] = 13.6$

$\dfrac{hc}{\wedge_n} = 13.6 - \dfrac{13.6}{n^2} = 13.6\left[\dfrac{n^2-1}{n^2}\right]$

$\wedge_n = \dfrac{hc}{13.6}\cdot\dfrac{n^2}{n^2-1} = \dfrac{hc}{13.6} + \dfrac{hc}{13.6(n^2-1)}$

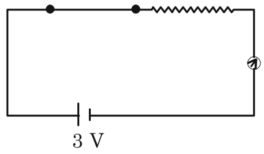
$\wedge_n \approx \dfrac{hc}{13.6} + \dfrac{hc}{13.6n^2}$

$\Rightarrow \wedge_n = A + \dfrac{hcK_2^2}{13.6\lambda_n^2} = A + B/\lambda_n^2$

63. (2)

Since the diode is non-ideal so there will be a voltage drop of 0.7V across it.

$i = \dfrac{3 - 0.7}{200} \times 10^3 = 11.5$ mA



64. (2)

$P = \dfrac{m}{V} = \dfrac{m}{\ell^3}$

$\dfrac{\Delta P}{P} \times 100 = \dfrac{\Delta m}{m} \times 100 + 3\dfrac{\Delta \ell}{\ell} \times 100$

$= 1.5 + 3 \times 1$

$\dfrac{\Delta P}{P} \times 100 = 4.5\%$

65. (1)

$r = \dfrac{mv}{qB} = \dfrac{mv^2}{qBv} = \dfrac{2k}{qBv}$

$\Rightarrow r\alpha \dfrac{1}{qv} \Rightarrow r = \dfrac{k}{qv}$

$m_\alpha = 4mp$

$m_e << mp$

As KE of all particles is equal

$V_e >> V_p$

$v_\alpha = {}^{v_p}/_2$

Let $q_e = 1$, $q_p = 1$, $q_\alpha = 2$

So $r_e = \dfrac{k}{v_e}$

$r_p = \dfrac{k}{v_p}$

$r_\alpha = \dfrac{k}{2 \times {}^{v_p}/_2} = \dfrac{k}{v_p}$

So, $r_p = r_\alpha$ and $r_e < r_p$ [$\because$ $v_e >> v_p$]

$\therefore$ Correct order is $r_e < r_p = r_\alpha$

66. (1)

$\sigma = \dfrac{q_A}{4\pi a^2}$

$q_A = \sigma 4\pi a^2$

$q_b = -\sigma 4\pi b^2$

$q_c = \sigma 4\pi c^2$

$$V_B = \frac{1}{4\pi \in 0}\sigma\left[\frac{4\pi a^2}{b} - \frac{q_B}{b} + \frac{q_C}{c}\right]$$

$$V_B = \frac{1}{4\pi \in 0}\sigma\left[\frac{4\pi a^2}{b} - \frac{4\pi b^2}{b} + \frac{4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\in 0}\left[\frac{a^2 - b^2}{b} + c\right]$$

67. (1)

$m_1 = 5$ kg

$m_2 = 10$ kg

$\mu = 0.15$

for stopping the motion.

$T = f_2 = \mu (m + m_2)g.$ ________ (1)

$T = m_1 g.$ ________(2)

$m_1 g = \mu (m_1 + m_2)$ g

$$m + m_2 = \frac{m_1}{\mu}$$

$$m = \frac{m_1}{\mu} - m_2 = \frac{5}{0.15} - 10 = \frac{500}{15} - 10$$

$m = 23.33$ kg.

$\therefore$ Minimum $m = 27.3$ kg

68. (2)

$$U = \frac{-k}{2r^2}$$

$$F = \frac{-dU}{dr} = +\frac{d}{dr}\left(\frac{k}{2r^2}\right) = \frac{k}{2}\frac{d}{dr}\left(r^{-2}\right)$$

$$F = \frac{k}{2}(-2)r^{-3} = \frac{-k}{r^3} = \frac{mv^2}{r}$$

$$K.E = \left|\frac{1}{2}mv^2\right| = \left|\frac{-k}{2r^2}\right| = \frac{k}{2r^2}$$

T.E = K.E + P.E

$$T.E = \frac{k}{2r^2} - \frac{k}{2r^2} = 0$$

69. (4)

$q_0 = C_0 V$

$q = KC_0 V$

Induced charge, $q_i = q - q_0 = C_0 V (K - 1)$

$= 90 \times 10^{-12} \times 20 \times 2/3$

$= 1200 \times 10^{-12} = 1.2$ nC

70. (1)

$\nu = 10^{12}$ Hz

$108 = 6.02 \times 10^{23}$m.

$$m = \frac{108 \times 10^{-3}}{6.02 \times 10^{23}} \text{kg}$$

$$\frac{1}{\nu} = T = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{1}{\nu^2} = 4\pi^2\frac{m}{k}$$

$k = 4\pi^2 m\nu^2$

$$k = \frac{4 \times 9.8596 \times 108 \times 10^{+24}}{6.02 \times 10^{23}}$$

$k = 7.07$ N/m.

71. (4)

$e = 1$, $v_2 = 0$, $u_1 = u$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u$$

$$\text{Fraction less of KE} = \frac{KEi - KE_f}{KE_i} = \frac{m_1 v^2 - m_1 v_1^2}{m_1 v^2}$$

$$= \frac{v^2 - v_1^2}{v^2} = \frac{(m_1 + m_2)^2 - (m_1 - m_2)^2}{(m_1 + m_2)^2} = \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

For deuterium, $m_1 = m$ and $m_2 = 2m$

$$Pd = \frac{4 \times m \times 2m}{(m + 2m)^2} = \frac{8}{9} = 0.89$$

For carbon, $m_1 = m$ and $m_2 = 12m$

$$P_c = \frac{4 \times m \times 12m}{(m + 12m)^2} = \frac{48}{169} = 0.28$$

72. (2)

$M = iA = \pi r^2 A \Rightarrow M \propto r^2$

$$B = \frac{\mu_0 i}{2r} \Rightarrow B \propto \frac{1}{r}$$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{M_1}{M_2}}$$

$$\text{Now, } \frac{M_1}{M_2} = \frac{r_1^2}{r_2^2} = \frac{1}{2}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{B_2}{B_2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{B_1}{B_2} = \sqrt{2}$$

73. (1)

Let us assume voltage gradient of **wire** = K

Now emt of cell, e = 52k ___________ (i)

Terminal voltage of cell, v = 40k

$$v = e - ir = e\left\{1 - \frac{r}{R + r}\right\} = 40\text{ k}$$

$\Rightarrow 52k\left[\frac{R}{R+r}\right] = 40\ k$

$\Rightarrow \frac{R}{R+r} = \frac{40}{52} = \frac{10}{13}$

$\Rightarrow \frac{5}{5+r} = \frac{10}{13} \Rightarrow r = 1.5\ \Omega$

74. (2)

10% of 10 6thr $= 10 \times 10 \times \frac{10}{100} = 109\ \text{Hz}$

No. of channels $= \frac{10^9}{5\times10^3} = \frac{10^6}{5} = 2\times10^5$

75. (2)

$2^1 = \frac{2_0}{2}\cos^2\theta$

$\frac{2_0}{8} = \frac{2_0}{2}\cos^2\theta = \cos^2\theta$

$\frac{1}{4} = \cos^2\theta$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 45°$

76. (2)

$\frac{R_1}{\ell} = \frac{R_2}{100-\ell}$

$\frac{R_2}{\ell-10} = \frac{R_1}{110-\ell}$

$R_1 + R_2 = 1000$

Solving these equations, we have

$R_1 = 550\Omega$ & $R_2 = 450\Omega$.

77. (4)

Mass of $\pi R^2 = 9M$

,, ,, 1 Area $= \frac{9M}{\pi R^2}$

Mass of $\pi\left(\frac{R^2}{9}\right) = \frac{9M}{\pi R^2}\times\pi\frac{R^2}{9} = \frac{9M}{9} = M.$

Remaining moment of inertia

$I = I_{total} - I_{hole}$

$I = \frac{9MR2}{2} - \frac{MR^2}{2}$

$I_{hole} = \frac{M\left(\frac{R}{3}\right)^2}{2} + M\left(\frac{2R}{3}\right)^2$

$I_{hole} = \frac{MR^2}{9\times2} + \frac{4r^2MR^2}{9\times2}$

$= \frac{5}{9}\frac{MR^2}{2}$

$I = 4MR^2$

78. (1)

$\frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 = \frac{150}{100}\times\frac{1}{2}mv_0^2$

$v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \rightarrow (1)$

$mv_0 = -mv_1 + mv_2$

$v_0 = v_2 - v_1 \rightarrow (2)$

Solving these 2 equations, we have

$v_1 + v_2 = \sqrt{2}\,v_0$

79. (2)

$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c}-t\right)\right]$

$\vec{E}_1 = E_{01}\hat{x}\cos\left[\left(\frac{2\pi c}{\lambda}\right)\left(\frac{z}{c}-t\right)\right]$

$\vec{E}_1 = E_{01}\hat{x}\cos\left[k(z-ct)\right]$

$k_1 = k.$

$\vec{E}_2 = E_{02}\hat{x}\cos\left[k2(z-\frac{c}{2}t)\right]$

$k_2 = 2k$

we know that

$k = \sqrt{\epsilon_r}$

$\frac{\epsilon_{r1}}{\epsilon_{r2}} = \left(\frac{k_1}{k_2}\right)$

$\frac{\epsilon_{r1}}{\epsilon_{r2}} = \left(\frac{k_1}{2k}\right)^2 = \frac{1}{4}$

80. (4)

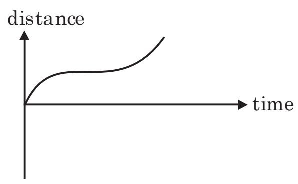
Quality Factor

$Q = \frac{\omega_0 L}{R}$

81. (1)

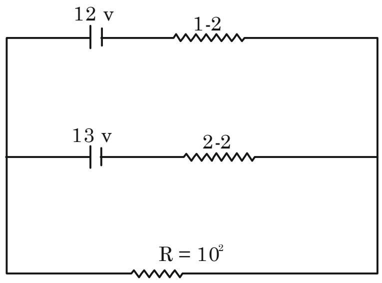
If V vs time is a straight line with –ve slope, Acceleration = –ve (constant)

Then displacement vs time will be parabola like



Obiviously incorrect option is (1).

82. (1)



$$\frac{\sum\left(\frac{E}{r}\right)}{\sum\frac{1}{r}} = \text{Net Emf.}$$

$$\text{Net Emf} = \frac{\frac{12}{1}+\frac{13}{2}}{\left(\frac{1\times2}{1\times2}\right)\frac{3}{2}} = \frac{37}{3}$$

$$I = \frac{\left(\frac{37}{3}\right)}{\frac{2}{3}+10} = \frac{37}{3}\times\frac{3}{32} = \frac{37}{32}\text{A.}$$

$V_R = IR$

$$V_R = \frac{37}{32}\times10 = 11.56\,\text{volt}$$

Max voltage = 11.56 which lies in only option (1).

83. (2)

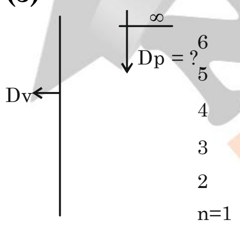
$$F \propto \frac{1}{R^2}$$

$$\frac{mv^2}{R} = \frac{k}{R^n} \Rightarrow \frac{mv^2}{R^2} = m\omega^2 = \frac{k}{R^{n+1}}$$

$$\frac{m4\pi^2}{T^2} = \frac{k}{R^{n+1}}$$

$$\Rightarrow T \propto R^{(n+1)/2}$$

84. (3)



Lyman $\frac{1}{\lambda} = \frac{v}{c}$

$$\frac{1}{\lambda_v} = R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]200$$

$$\frac{v_1}{C} = R.C\left[\frac{1}{1}\right]$$

$RC = v_C$ __________(i)

Phund

$$\frac{v_P}{C} = \frac{1}{\lambda_p} = R\left[\frac{1}{(5)^2}-\frac{1}{\infty}\right]$$

$$v_P = \frac{v_L}{25}$$

85. (1)

$$i_{rms} = \frac{20}{\sqrt{2}}$$

walters current = $i_{rms}\sin\phi$

$$= \frac{20}{\sqrt{2}}\times\frac{1}{\sqrt{2}}$$

= 10 A

power = $v_{rms}\, i_{rms} \cos\theta$

$$\frac{100}{\sqrt{2}}\times\frac{20}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$$

86. (2)

$n = 2$, $V_1 = V$, $T_1 = 27 + 273 = 300$ K

$\gamma = \frac{5}{3}$ (for monoatomic gas) $V_2 = 2V$, $T_2 = ?$

$$TV^{\gamma-1} = \text{Constant}$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\left(\frac{V}{2V}\right)^{\frac{5}{3}-1}$$

$$T_2 = 300\left(\frac{1}{2}\right)^{\frac{2}{3}}$$

$$T_2 \approx 189\text{K}$$

$$dQ = dv + dw$$

$$dv = -dw$$

$$dv = -\frac{1}{\gamma-1}(P_1V_1 - P_2V_2)$$

$$dv = -\frac{nR}{\gamma-1}(300-189)$$

$$dv = -\frac{2\times8.3}{0.6}\times111 = -2.7\text{ kJ}$$

87. (2)

$$K = \frac{\Delta P}{\Delta V / V}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta P}{K} = \frac{mg}{Ka}$$

$$\Rightarrow \frac{dr}{r} = \frac{1}{3}\frac{\Delta V}{V} = \frac{mg}{3Ka}$$

88. (4)

$v = \sqrt{\frac{y}{\rho}} = \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^{3}}}$

$\nu = \frac{1}{1.2}\sqrt{\frac{y}{\rho}}$

$\nu = \frac{1}{1.2}\sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^{3}}}$

$\nu = 4.85 \times 10^{3}$

$\nu \approx 5 khz$

89. (4)

$F = \Delta p / 1 \text{ sec} = 2mv\cos\theta \times n$

$P = \frac{F}{A} = \frac{2mvn\cos\theta}{A} = \frac{2 \times 3.32 \times 10^{-27} \times 10^{3} \times 10^{23}}{2 \times 10^{-4} \times \sqrt{2}}$

$= 2.35 \times 10^{3}$ N/m^2

90. (3)

$I' = \frac{MR^2}{2} + M(2R)^2$

$I' = \frac{MR^2}{2} + 4MR^2 = \frac{9}{2}MR^2$

$I = \frac{MR^2}{2} + 6 \times \frac{9}{2}MR^2$

$I = \frac{55}{2}MR^2$

$I_p = \frac{55}{2}MR^2 + 7M \times (3R)^2$

$I_p = \frac{181}{2}MR^2$