PHYSICS

1. **(3)**

$e = 1,\ v_2 = 0,\ u_1 = u$

$v_1 = \dfrac{m_1 - m_2}{m_1 + m_2}u$

Fraction less of KE $= \dfrac{KEi - KE_f}{KE_i} = \dfrac{m_1v^2 - m_1v_1^2}{m_1v^2}$

$= \dfrac{v^2 - v_1^2}{v^2} = \dfrac{(m_1+m_2)^2 - (m_1-m_2)^2}{(m_1+m_2)^2} = \dfrac{4m_1m_2}{(m_1+m_2)^2}$

For deuterium, $m_1 = m$ and $m_2 = 2m$

$Pd = \dfrac{4 \times m \times 2m}{(m+2m)^2} = \dfrac{8}{9} = 0.89$

For carbon, $m_1 = m$ and $m_2 = 12m$

$P_c = \dfrac{4 \times m \times 12m}{(m+12m)^2} = \dfrac{48}{169} = 0.28$

2. **(3)**

$F = \Delta p / 1\text{ sec} = 2mv\cos\theta \times n$

$P = \dfrac{F}{A} = \dfrac{2mvn\cos\theta}{A} = \dfrac{2 \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23}}{2 \times 10^{-4} \times \sqrt{2}}$

$= 2.35 \times 10^3\ N/m^2$

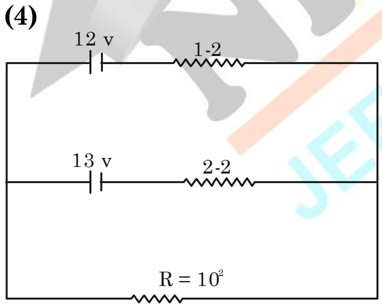
3. **(1)**

$K = \dfrac{\Delta P}{\Delta V / V}$

$\Rightarrow \dfrac{\Delta V}{V} = \dfrac{\Delta P}{K} = \dfrac{mg}{Ka}$

$\Rightarrow \dfrac{dr}{r} = \dfrac{1}{3}\dfrac{\Delta V}{V} = \dfrac{mg}{3Ka}$

4. **(4)**



$\dfrac{\sum\left(\dfrac{E}{r}\right)}{\sum\dfrac{1}{r}}$ = Net Emf.

Net Emf $= \dfrac{\dfrac{12}{1} + \dfrac{13}{2}}{\left(\dfrac{1\times2}{1\times2}\right)\dfrac{3}{2}} = \dfrac{37}{3}$

$I = \dfrac{\left(\dfrac{37}{3}\right)}{\dfrac{2}{3} + 10} = \dfrac{37}{3} \times \dfrac{3}{32} = \dfrac{37}{32}A.$

$V_R = IR$

$V_R = \dfrac{37}{32} \times 10 = 11.56\text{ volt}$

Max voltage = 11.56 which lies in only option (3).

5. **(1)**

$U = \dfrac{-k}{2r^2}$

$F = \dfrac{-dU}{dr} = +\dfrac{d}{dr}\left(\dfrac{k}{2r^2}\right) = \dfrac{k}{2}\dfrac{d}{dr}\left(r^{-2}\right)$

$F = \dfrac{k}{2}(-2)r^{-3} = \dfrac{-k}{r^3} = \dfrac{mv^2}{r}$

$K.E = \left|\dfrac{1}{2}mv^2\right| = \left|\dfrac{-k}{2r^2}\right| = \dfrac{k}{2r^2}$

$T.E = K.E + P.E$

$T.E = \dfrac{k}{2r^2} - \dfrac{k}{2r^2} = 0$

6. **(4)**

$m_1 = 5$ kg

$m_2 = 10$ kg

$\mu = 0.15$

for stopping the motion.

$T = f_2 = \mu (m + m_2)g.$ ________ (1)

$T = m_1g.$ ________(2)

$m_1g = \mu (m_1 + m_2)\ g$

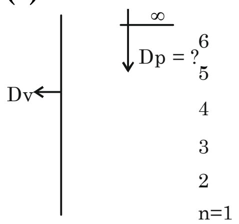
$m + m_2 = \dfrac{m_1}{\mu}$

$m = \dfrac{m_1}{\mu} - m_2 = \dfrac{5}{0.15} - 10 = \dfrac{500}{15} - 10$

$m = 23.33$ kg.

$\therefore$ Minimum $m = 27.3$ kg

7. **(2)**



Lyman $\dfrac{1}{\lambda} = \dfrac{v}{c}$

$\dfrac{1}{\lambda_v} = R\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]200$

$\dfrac{v_1}{C} = R.C\left[\dfrac{1}{1}\right]$

$RC = v_C$ ____________(i)

Phund

$\dfrac{v_P}{C} = \dfrac{1}{\lambda_p} = R\left[\dfrac{1}{(5)^2} - \dfrac{1}{\infty}\right]$

$v_P = \dfrac{v_L}{25}$

8. **(1)**

$$2^1 = \frac{2_0}{2}\cos^2\theta$$

$$\frac{2_0}{8} = \frac{2_0}{2}\cos^2\theta = \cos^2\theta$$

$$\frac{1}{4} = \cos^2\theta$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45°$$

9. **(3)**

K.E. of $\bar{e}$ in n^{th} orbit $= \frac{13.6}{n^2} = \frac{1}{2}\frac{p^2}{m_e}$

$$\Rightarrow P = \frac{\sqrt{27.2me}}{n} = \frac{k_1}{n} \quad \{\text{Let say } K_1 = \sqrt{27.2me}$$

Now $\lambda_n = \frac{h}{p} = \frac{hn}{k_1} \Rightarrow \lambda_n = k_2 n$

$$\frac{h}{\lambda n} = 13.6\left[1 - \frac{1}{n^2}\right] = 13.6$$

$$\frac{hc}{\wedge_n} = 13.6 - \frac{13.6}{n^2} = 13.6\left[\frac{n^2-1}{n^2}\right]$$

$$\wedge_n = \frac{hc}{13.6}\cdot\frac{n^2}{n^2-1} = \frac{hc}{13.6} + \frac{hc}{13.6(n^2-1)}$$

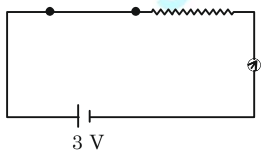
$$\wedge_n \approx \frac{hc}{13.6} + \frac{hc}{13.6n^2}$$

$$\Rightarrow \wedge_n = A + \frac{hcK_2^2}{13.6\lambda_n^2} = A + B/\lambda_n^2$$

10. **(1)**

Since the diode is non-ideal so there will be a voltage drop of 0.7V across it.

$$i = \frac{3-0.7}{200}\times 10^3 = 11.5 \text{ mA}$$



11. **(3)**

$$r = \frac{mv}{qB} = \frac{mv^2}{qBv} = \frac{2k}{qBv}$$

$$\Rightarrow r \alpha \frac{1}{qv} \Rightarrow r = \frac{k}{qv}$$

$m_\alpha = 4mp$

$m_e << mp$

As KE of all particles is equal

$V_e >> V_p$

$v_\alpha = v_p/2$

Let $q_e = 1$, $q_p = 1$, $q_\alpha = 2$

So $r_e = \frac{k}{v_e}$

$r_p = \frac{k}{v_p}$

$$r_\alpha = \frac{k}{2\times v_p/2} = \frac{k}{v_p}$$

So, $r_p = r_\alpha$ and $r_e < r_p$ [$\because$ $v_e >> v_p$]

$\therefore$ Correct order is $r_e < r_p = r_\alpha$

12. **(3)**

$q_0 = C_0V$

$q = KC_0V$

Induced charge, $q_i = q - q_0 = C_0V(K-1)$

$= 90 \times 10^{-12} \times 20 \times 2/3$

$= 1200 \times 10^{-12} = 1.2$ nC

13. **(3)**

Quality Factor

$$Q = \frac{\omega_0 L}{R}$$

14. **(1)**

$$10\% \text{ of } 10\ 6\text{thr} = 10\times 10\times\frac{10}{100} = 109 \text{ Hz}$$

$$\text{No. of channels} = \frac{10^9}{5\times 10^3} = \frac{10^6}{5} = 2\times 10^5$$

15. **(3)**

$$v = \sqrt{\frac{y}{\rho}} = \sqrt{\frac{9.27\times 10^{10}}{2.7\times 10^3}}$$

$$v = \frac{1}{1.2}\sqrt{\frac{y}{\rho}}$$

$$v = \frac{1}{1.2}\sqrt{\frac{9.27\times 10^{10}}{2.7\times 10^3}}$$

$$v = 4.85\times 10^3$$

$$v \approx 5\text{khz}$$

16. **(2)**

$$I' = \frac{MR^2}{2} + M(2R)^2$$

$$I' = \frac{MR^2}{2} + 4MR^2 = \frac{9}{2}MR^2$$

$$I = \frac{MR^2}{2} + 6\times\frac{9}{2}MR^2$$

$$I = \frac{55}{2}MR^2$$

$$I_p = \frac{55}{2}MR^2 + 7M \times (3R)^2$$

$$I_p = \frac{181}{2}MR^2$$

17. (4)

$$\sigma = \frac{q_A}{4\pi a^2}$$

$$q_A = \sigma 4\pi a^2$$

$$q_b = -\sigma 4\pi b^2$$

$$q_c = \sigma 4\pi c^2$$

$$V_B = \frac{1}{4\pi \in 0}\sigma\left[\frac{4\pi a^2}{b} - \frac{q_B}{b} + \frac{q_C}{c}\right]$$

$$V_B = \frac{1}{4\pi \in 0}\sigma\left[\frac{4\pi a^2}{b} - \frac{4\pi b^2}{b} + \frac{4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\in 0}\left[\frac{a^2 - b^2}{b} + c\right]$$

18. (4)

Let us assume voltage gradient of **wire** = K

Now emt of cell, e = 52k ____________ (i)

Terminal voltage of cell, v = 40k

$$v = e - ir = e\left\{1 - \frac{r}{R+r}\right\} = 40\ k$$

$$\Rightarrow\ 52k\left[\frac{R}{R+r}\right] = 40\ k$$

$$\Rightarrow \frac{R}{R+r} = \frac{40}{52} = \frac{10}{13}$$

$$\Rightarrow \frac{5}{5+r} = \frac{10}{13} \Rightarrow r = 1.5\ \Omega$$

19. (1)

$$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$$

$$\vec{E}_1 = E_{01}\hat{x}\cos\left[\left(\frac{2\pi c}{\lambda}\right)\left(\frac{z}{c} - t\right)\right]$$

$$\vec{E}_1 = E_{01}\hat{x}\cos[k(z - ct)]$$

$k_1 = k$.

$$\vec{E}_2 = E_{02}\hat{x}\cos\left[k2(z - \frac{c}{2}t)\right]$$

$k_2 = 2k$

we know that

$$k = \sqrt{\in_r}$$

$$\frac{\in_{r1}}{\in_{r2}} = \left(\frac{k_1}{k_2}\right)$$

$$\frac{\in_{r1}}{\in_{r2}} = \left(\frac{k_1}{2k}\right)^2 = \frac{1}{4}$$

20. (3)

$2\theta = 60°$

$\theta = 30°$

$d\ \sin\theta = \lambda$, (d=width of single slit=1μ m = 10^{-6} m.)

$\lambda = 10^{-6}$ Sin30°

$\lambda = 0.5 \times 10^{-6}$

for Double Slit

$$\beta = \frac{D\lambda}{d}$$ (d = slit separation, D= 50 cm = 0.5 m)

$$10^{-2} = \frac{0.5 \times 0.5 \times 10^{-6}}{d}$$

$d = 25 \times 10^{-6}$ m.

$d = 25\ \mu$ m

21. (4)

$\nu = 10^{12}$Hz

$108 = 6.02 \times 10^{23}$m.

$$m = \frac{108 \times 10^{-3}}{6.02 \times 10^{23}}kg$$

$$\frac{1}{\nu} = T = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{1}{\nu^2} = 4\pi^2\frac{m}{k}$$

$$k = 4\pi^2 m\nu^2$$

$$k = \frac{4 \times 9.8596 \times 108 \times 10^{+24}}{6.02 \times 10^{23}}$$

k = 7.07 N/m.

22. (3)

Mass of $\pi R^2 = 9M$

,, ,, 1 Area = $\frac{9M}{\pi R^2}$

$$\text{Mass of } \pi\left(\frac{R^2}{9}\right) = \frac{9M}{\pi R^2} \times \pi\frac{R^2}{9} = \frac{9M}{9} = M.$$

Remaining moment of inertia

$I = I_{total} - I_{hole}$

$$I = \frac{9MR2}{2} - \frac{MR^2}{2}$$

$$I_{hole} = \frac{M\left(\frac{R}{3}\right)^2}{2} + M\left(\frac{2R}{3}\right)^2$$

$$I_{hole} = \frac{MR^2}{9 \times 2} + \frac{4r^2MR^2}{9 \times 2}$$

$$= \frac{5}{9}\frac{MR^2}{2}$$

$I = 4MR^2$

23. (4)

$$\frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 = \frac{150}{100} \times \frac{1}{2}mv_0^2$$

$$v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \rightarrow (1)$$

$$mv_0 = -mv_1 + mv_2$$

$$v_0 = v_2 - v_1 \rightarrow (2)$$

Solving these 2 equations, we have

$v_1 + v_2 = \sqrt{2}\, v_0$

24. (1)

$M = iA = \pi r^2 A \Rightarrow M \propto r^2$

$B = \frac{\mu_0 i}{2r} \Rightarrow B \propto \frac{1}{r}$

$\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{M_1}{M_2}}$

Now, $\frac{M_1}{M_2} = \frac{r_1^2}{r_2^2} = \frac{1}{2}$

$\Rightarrow \frac{r_1}{r_2} = \frac{B_2}{B_2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{B_1}{B_2} = \sqrt{2}$

25. (1)

$P = \frac{m}{V} = \frac{m}{\ell^3}$

$\frac{\Delta P}{P} \times 100 = \frac{\Delta m}{m} \times 100 + 3\frac{\Delta \ell}{\ell} \times 100$

$= 1.5 + 3 \times 1$

$\frac{\Delta P}{P} \times 100 = 4.5\%$

26. (1)

$\frac{R_1}{\ell} = \frac{R_2}{100 - \ell}$

$\frac{R_2}{\ell - 10} = \frac{R_1}{110 - \ell}$

$R_1 + R_2 = 1000$

Solving these equations, we have

$R_1 = 550\,\Omega$ & $R_2 = 450\,\Omega$.

27. (4)

$i_{rms} = \frac{20}{\sqrt{2}}$

walters current = $i_{rms} \sin\phi$

$= \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

= 10 A

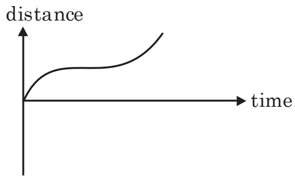
power = $v_{rms}\, i_{rms} \cos\theta$

$\frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$

28. (4)

If V vs time is a straight line with –ve slope, Acceleration = –ve (constant)

Then displacement vs time will be parabola like



Obiviously incorrect option is (3).

29. (1)

$n = 2$, $V_1 = V$, $T_1 = 27 + 273 = 300$ K

$\gamma = \frac{5}{3}$ (for monoatomic gas) $V_2 = 2V$, $T_2 = ?$

$TV^{\gamma-1} = \text{Constant}$

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\left(\frac{V}{2V}\right)^{\frac{5}{3}-1}$

$T_2 = 300\left(\frac{1}{2}\right)^{\frac{2}{3}}$

$T_2 \approx 189\text{K}$

$dQ = dv + dw$

$dv = -dw$

$dv = -\frac{1}{\gamma - 1}(P_1V_1 - P_2V_2)$

$dv = -\frac{nR}{\gamma - 1}(300 - 189)$

$dv = -\frac{2 \times 8.3}{0.6} \times 111 = -2.7\text{ kJ}$

30. (1)

$F \propto \frac{1}{R^2}$

$\frac{mv^2}{R} = \frac{k}{R^n} \Rightarrow \frac{mv^2}{R^2} = m\omega^2 = \frac{k}{R^{n+1}}$

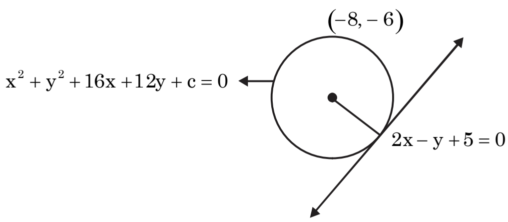
$\frac{m4\pi^2}{T^2} = \frac{k}{R^{n+1}}$

$\Rightarrow T \propto R^{(n+1)/2}$

MATHEMATICS

31. (2)

Tangent at (1, 7) to $x^2 = y - 6$ is $2x - y + 5 = 0$



Now, $\left|\dfrac{-16+6+5}{\sqrt{5}}\right| = \sqrt{64+36-C}$

$\Rightarrow C = 95$

32. (4)

Plane passing through 2x – 2y + 3z –2 = 0 any x – y + z + 1 = 0 is given by,

$(2x-2y+3z-2) + \lambda(x-y+z+1) = 0$

$\Rightarrow (\lambda+2)x - (\lambda+2)y + (\lambda+3)z + (\lambda-2) = 0$ ...(1)

Plane (1) and x + 2y – z – 3 = 0 and 3x – y + 2z – 1 = 0 has infinitely many solution.

$$\Rightarrow \begin{vmatrix} \lambda+2 & -(\lambda+2) & \lambda+3 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda = 5$

Equation of required plane is

$7x - 7y + 8z + 3 = 0$

Distance from origin $= \dfrac{3}{\sqrt{49+49+64}}$

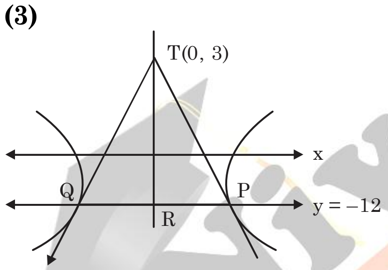
$= \dfrac{3}{9\sqrt{2}} = \dfrac{1}{3\sqrt{2}}$

33. (1)

$\alpha = -\omega,\ \beta = -\omega^2$

$\therefore\ \alpha^{101} + \beta^{107} = -(\omega^2+\omega) = 1$

34. (3)



Area $= \dfrac{1}{2} \times PA \times TR$ $\quad \begin{cases} TR = 15 \\ PQ = 6\sqrt{5} \end{cases}$

$\dfrac{1}{2} \times 15 \times 6\sqrt{5}$

$= 45\sqrt{5}$

35. (2)

from $y^2 = 6x$, $\dfrac{dy}{dx} = \dfrac{3}{y}$

from $9x^2 + by^2 = 16$, $\dfrac{dy}{dx} = \dfrac{-ax}{by}$

ATQ, Curves cut at right angle,

$\dfrac{-ax}{by} \times \dfrac{3}{y} = -1 \Rightarrow by^2 = 27x \Rightarrow b = \dfrac{27x}{y^2} = \dfrac{27x}{6x} = \dfrac{9}{2}$

36. (4)

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0 \Rightarrow k = 11$$

$x + 11y + 3z = 0$ ______ (i)

$3x + 11y - 2z = 0$ ______ (ii)

$2x + 4y - 3z = 0$ ______ (iii)

+ (ii) $\Rightarrow x = -5y$

putting it in (i), we get

$-5y + 11y + 3z = 0$

$\Rightarrow z = -2y$

so, $\dfrac{xz}{y^2} = \dfrac{(-5y)(-2y)}{y^2} = 10$

37. (1)

Case – 1 :

$2(3-\sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 4, 2$

$\Rightarrow x = 16$ or $x = 4$ (Rejected)

Case – 2 :

Let $x \in [9, \infty)$

$2(\sqrt{x}-3) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 16$ or $x = 0$ (Rejected)

So, x = 4, 16 are two solutions.

38. (4)

$8\cos x\left\{\cos\left(\dfrac{\pi}{6}+x\right)\cos\left(\dfrac{\pi}{6}-x\right) - \dfrac{1}{2}\right\} = 1$

$\Rightarrow \cos x\left\{\cos^2 x - \sin^2\dfrac{\pi}{6} - \dfrac{1}{2}\right\} = \dfrac{1}{8}$

$\Rightarrow \cos x\left(\cos^2 x - \dfrac{3}{4}\right) = \dfrac{1}{8}$

$\Rightarrow \dfrac{4\cos^3 x - 3\cos x}{4} = \dfrac{1}{8}$

$\Rightarrow \cos 3x = \dfrac{1}{2} = \cos\dfrac{\pi}{3}$

$\Rightarrow 3x = 2n\pi \pm \dfrac{\pi}{3} \Rightarrow x = \dfrac{2n\pi}{3} \pm \dfrac{\pi}{9}$

In $[0, \pi]$ sum of solutions $= \dfrac{\pi}{9} + \dfrac{2\pi}{3} + \dfrac{\pi}{9} + \dfrac{2\pi}{3}$

$= \dfrac{3\pi}{9}$

$K = \dfrac{13}{9}$

39. (4)

Required probability

$$P(R_1)\,P\left(\frac{R_2}{R_1}\right) + P(B_1) \times P\left(\frac{R_2}{B_1}\right)$$

$$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12}$$

R_1, R_2 are drawing red balls in Ist and 2nd draw
B_1 = drawing black ball in Ist draw.

$$= \frac{2}{5}$$

40. (2)

$$h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \frac{\left(x - \frac{1}{x}\right)^2 + 2}{x - \frac{1}{x}}$$

$$= \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}$$

We know, $\dfrac{\left(x - \frac{1}{x}\right) + \dfrac{2}{\left(x - \frac{1}{x}\right)}}{2} \geq \sqrt{\left(x - \frac{1}{x}\right) \times \dfrac{2}{\left(x - \frac{1}{x}\right)}}$

$$\Rightarrow \left(x - \frac{1}{x}\right) + \frac{2}{x - \frac{1}{x}} \geq 2\sqrt{2}$$

$\Rightarrow$ minimum value of h(x) = $2\sqrt{2}$

41. (4)

Set A contains all parts inside

$|x| < 1$ and $|y| < 1$

Set B contains all parts inside the ellipse all parts inside that ellipse $\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$

clearly ACB.

42. (3)

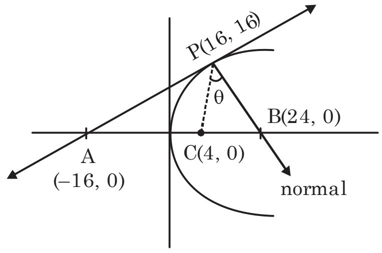
$\sim (p \vee g) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p \wedge (\sim q \vee q)$

$= \sim p \wedge t = \sim p$

43. (4)



equation of tangent is
$x - 2y + 16 = 0$

$m_{PC} = \frac{4}{3}$

$m_{PB} = -2.$

$$\therefore \tan\theta = \left|\frac{\frac{4}{3} + 2}{1 + \frac{4}{3} \times (-2)}\right| = 2$$

44. (1)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (5x-4)(x+4)^2$$

so, eqating, A = –4, B = 5

45. (2)

Let $\sqrt{x^2 - 1} = a$

We have, $(x + a)^5 + (x - a)^5$

$= 2\left[{}^5C_0 x^5 + {}^5C_2 x^3 a^2 + {}^5C_4 x^5 a^4\right]$

$= 2[x^5 + 10x^3 (x^3 - 1) + 5x (x^3 - 1)^2]$

$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

Considering odd degree terms,

$2\,[x^5 + 5x^7 - 10x^3 + 5x]$

Sum of coefficients = 2

46. (1)

$$\sum_{k=0}^{12} a_{4k+1} = 416$$

$$\Rightarrow \frac{13}{2}(2a_1 + 48d) = 416$$

$\Rightarrow a_1 + 24d = 31$ ...(i)

$a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50_d = 66$ ...(ii)

from (i) and (ii), d = 1, $a_1 = 8$

Now, $140\,m = \sum_{r=1}^{17} ar^2$

$$= \sum_{r=1}^{17} \left[8 - 1(r-1)\cdot 1\right]^2$$

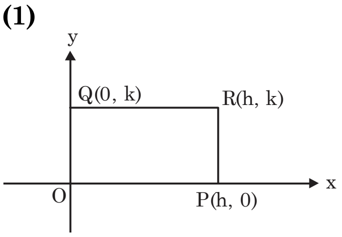
$$= \sum_{r=1}^{17} (r+7)^2$$

$$= \sum_{r=1}^{24} r^2 - \sum_{r=1}^{7} r^2$$

$$= \frac{24 \times 25 \times 49}{6} - \frac{7 \times 8 \times 15}{6}$$

$\Rightarrow$ $m = 34$

47. (1)



Equation in PQ is

$$\frac{x}{h} = \frac{y}{k} = 1$$

Putting (2, 3), we get $\frac{2}{h} + \frac{3}{k} = 1$

$\therefore$ Locus will be,

$$\frac{2}{x} + \frac{3}{y} = 1$$

$\Rightarrow 3x + 2y = xy$

48. (2)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$$

$$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{2^x \sin^2 x}{1 + 2^x} dx$$

$$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} \sin^2 x \, dx = 2\int_0^{\pi/2} \sin^2 x dx$$

$$\Rightarrow I = \int_0^{\pi/2} \sin^2 dx = \int_0^{\pi/2} \cos^2 x \, dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$$

49. (3)

$18x^2 - 9\pi x + \pi^2 = 0$

$$\Rightarrow x = \frac{9\pi \pm 3\pi}{36} \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}$$

i.e. $\alpha = \frac{\pi}{6}$, $\beta = \frac{\pi}{3}$

$$\text{Area} = \int_{\pi/6}^{\pi/3} gof(x) \, dx$$

$$= \int_{\pi/6}^{\pi/3} \cos x \, dx = [\sin x]_{\pi/6}^{\pi/3} = \frac{1}{2}\left(\sqrt{3} - 1\right)$$

50. (1)

Let $f(x) = x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

$$= x\left(\frac{1}{x} + \frac{2}{x} + ... + \frac{15}{x} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right)\right)$$

$$= x\left(\frac{15 \times 16}{2x} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right)\right)$$

$$= 120 - x\left\{\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right\}$$

We know $0 \le \left\{\frac{v}{x}\right\} < 1$

$$\Rightarrow 0 \le x\left\{\frac{v}{x}\right\} < x$$

$$\Rightarrow \lim_{x \to 0^+} x\left\{\frac{v}{x}\right\} = 0$$

$$\Rightarrow \lim_{x \to 0} f(x) = 120$$

51. (1)

$$\sum_{i=0}^{9}(x_i - 5) = 9 \Rightarrow \sum_{i=1}^{9} x_i = 54$$

$$\text{Again} \sum_{i=1}^{9}(x_i = 5)^2 = 45$$

$$\Rightarrow \sum_{i=1}^{9} x_i^2 - 10\sum_{i=1}^{9} x_i + 225 = 45$$

$$\Rightarrow \sum_{i=1}^{9} x_i^2 - 360$$

$$\text{Variance} = \sum_{i=1}^{9} x_i^2 - \left(\frac{\sum_{i=1}^{9} x_i}{9}\right)^2$$

$$= \frac{360}{9} - \left(\frac{54}{9}\right)^2$$

$= 4$

$\Rightarrow$ S.D $= 2$

52. (4)

$$\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + 1 + \cos^5 x\right)^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{\cos^{10} x\left(\tan^5 x + \tan^2 x + \tan^3 x + 1\right)} dx$$

$$= \int \frac{\frac{\sin^2 x}{\cos^2 x} \times \sec^6 x}{\left(\tan^3 x + 1\right)^2 \left(\tan^2 x + 1\right)^2} dx$$

$= \int \frac{\tan^2 x - \sec^6 x}{(\tan^3 x + 1)^2 \cdot \sec^4 x} dx$

$= \int \frac{\tan^2 x - \sec^2 x}{(1 + \tan^3 x)^2} dx$

Putting $1 + \tan^3 x = t \Rightarrow dt = 3\tan^2 x \sec^2 x \, dx$

$= \frac{1}{3}\int \frac{1}{t^2} dt = -\frac{1}{3t} + c$

$= -\frac{1}{3(1 + \tan^3 x)} + c$

53. (3)

$f(x) = |x - \pi| \left(e^{|x|} - 1\right) \sin|x|$

at $x = 0$

L.H.D = 0, R.H.d = 0

$\Rightarrow$ f(x) is differentiable.

$\therefore S = \phi$

54. (1)

$\sin x \, dy + y \cos x \, dx = 4x \, dx$

$\Rightarrow d(y \sin x) = 4x \, dx$

Integrating, $y \sin x = 2x^2 + c$

Curve passes through $\left(\frac{\pi}{2}, 0\right)$

$\Rightarrow 0 = \frac{\pi^2}{2} + c \Rightarrow c = -\frac{\pi^2}{2}$

Now, $y \sin x = 2x^2 - \frac{\pi^2}{2}$

$\therefore y\left(\frac{\pi}{6}\right) = -\frac{8}{9}\pi^2$

55. (3)

Let $\vec{u} = x\vec{a} + y\vec{b}$

Now $\vec{u} = \vec{a} \cdot \vec{b} = 0 \Rightarrow 14x + 2y = 0 \Rightarrow y = -7x$ ....(i)

$(\because |\vec{a}|^2 = 14, |\vec{b}|^2 = 2)$

$\vec{u} \cdot \vec{b} = 24 \Rightarrow 2x + 2y = 24$ $(\because \vec{a} \cdot \vec{b} = 2)$

$\Rightarrow x + y = 12$ ......(ii)

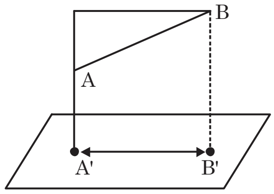
From (i) & (ii) $x = -2, y = 14$

$\therefore \vec{u} = -2(2\hat{i} + 3\hat{j} - \hat{k}) + 14(\hat{j} + \hat{k}) = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow |\vec{u}|^2 = 336$

56. (2)

Let A = (4, –1, 3), B = (5, –1, 4)

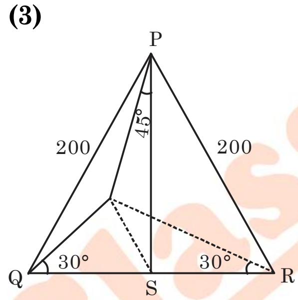


$AC = \overline{AB} \cdot \widehat{AC} = (\hat{i} + \hat{k}) \cdot \left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{\frac{2}{3}}$

$\Rightarrow$ Length of projection $= \sqrt{\frac{2}{3}}$

57. (3)



Let height of tower

ST = h

In Δ QST, $\tan 30° = \frac{ST}{QS}$

$\Rightarrow QS = \sqrt{3}h = SR$

In Δ STP, ST = PS

In Δ PSQ, $PS = \sqrt{(200)^2 - (\sqrt{3}h)^2}$

So, $\sqrt{(200)^2 - 3h^2} = h \Rightarrow h = 100m$

58. (3)

6 3

no. ways $= 6 \times 6 \times 41$

4 1

= 1080

59. (4)

$B - 2A = \sum_{r=1}^{40} t_r - 2\sum_{r=1}^{20} t_r$

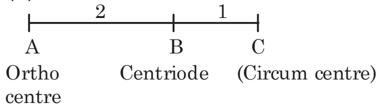
$(21^2 + 2.22^2 + ... + 40^2) - (1^2 + 2.2^2 + ... + 20^2)$

= 20[(22 + 24 + ...+ 60) + (24 + 28 + ...+60)]

$= 20\left[\frac{20}{2}(22 + 60) + \frac{10}{2}(24 + 60)\right]$

$100\lambda = 100 \times 248 \Rightarrow \lambda = 248$

60. (1)



$AC = \frac{3}{2}AB = \frac{3}{2} \times 2\sqrt{10} = 3\sqrt{10}$

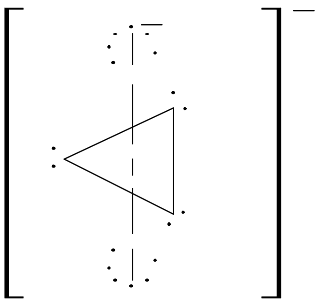
Radius of circle with AC as diameter

$= \frac{3}{2}\sqrt{10} \quad = 3\sqrt{\frac{5}{2}}$

CHEMISTRY

61. (1)

Structure of I_3^-

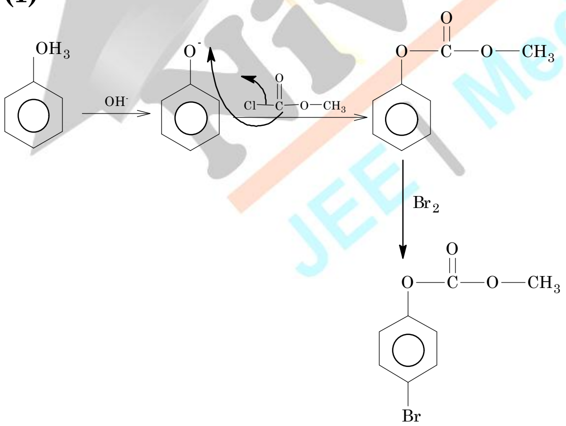


Number of lone pairs in $I_3^{\ominus}$ is 9.

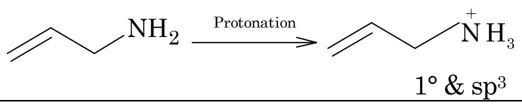
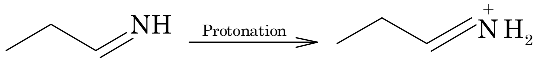
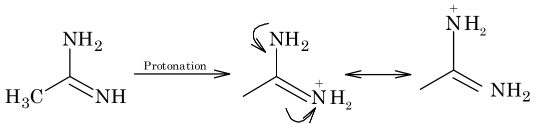
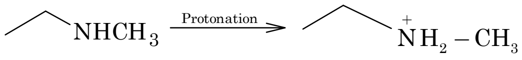
62. (4)

$CH_3COOK + H_2O \rightarrow CH_3COOH + KOH$ Basic
$FeCl_3$ – Acidic solution
$Al(CN)_3$ – Salt of weak acid and weak base
$Pb(CH_3COO)_2$ – Salt of weak acid and weak base
CH_3COOK is salt of weak acid and strong base.
Hence solution of CH_3COOK is basic.

63. (1)



64. (1)

(a)	
(b)	
	sp²
(c)	 [Equivalent resonance]
(d)	 2° & sp³

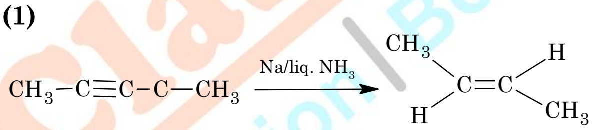
$\therefore$ Correct order of basically : b < a < d < c.

65. (1)

$\longleftarrow$ 3.9 — 4.5 $\longrightarrow$
Pinkish red   Yellow

Weak base is having pH greater than 7. When methyl orange is added to weak base solution, the solution becomes yellow. This solution is titrated by strong acid and at the end point pH will be less than 3.1. Therefore solution becomes pinkish red.

66. (1)



So, option (4) is correct.

67. (2)

Element	Relative mass	Relative mole	Simplest whole number ratio
C	6	$\frac{6}{12} = 0.5$	1
H	1	$\frac{1}{1} = 1$	2

So, X = 1, Y = 2

Equation for combustion of C_XH_Y

$$C_XH_Y + \left(X + \frac{Y}{4}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

Oxygen atoms required $= 2\left(X + \frac{Y}{4}\right)$

As per information,

$$2\left(X + \frac{Y}{4}\right) = 2Z$$

$$\Rightarrow \left(1 + \frac{2}{4}\right) = Z$$

$$\Rightarrow Z = 1.5$$

Molecule can be written

$C_XH_YO_Z$

$C_1H_2O_{3/2}$

$\Rightarrow C_2H_4O_3$

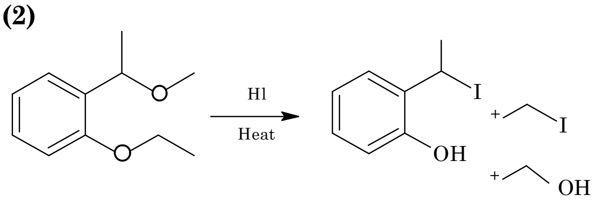
68. (1)

$[Fe(CN)_6]^{4-} + \frac{1}{2} H_2O_2 + H^+ \rightarrow [Fe(CN)_6]^{3-} + H_2O$

$[Fe(CN)_6]^{3-} + \frac{1}{2} H_2O_2 + OH^{\ominus} \rightarrow$

$[Fe(CN_6)^{4-}] + H_2O + \frac{1}{2} O_2$

69. (2)



Hence, option (2) is correct.

70. (1)

$B_2H_6 + 3O_2 \rightarrow B_2O_3 + 3H_2O$

27.66 of B_2H_6 = 1 mole of B_2H_6 which requires three moles of oxygen (O_2) for complete burning

$6H_2O \rightarrow 6H_2 + 3O_2$ (On electrolysis)

Number of faradays = 12 = Amount of charge

$12 \times 96500 = i \times t$

$12 \times 96500 = 100 \times t$

$t = \frac{12 \times 96500}{100}$ second

$t = \frac{12 \times 96500}{100 \times 3600}$ hour $\Rightarrow t = 3.2$ hours

71. (3)

Equilibrium constant $K = \left(\frac{A_t}{A_b}\right) e^{-\frac{\Delta H^\circ}{RT}}$

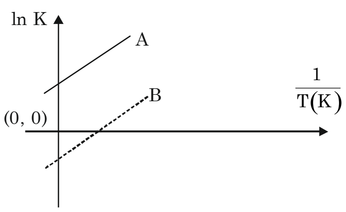
$\ln K = \ln \left(\frac{A_t}{A_b}\right) - \frac{\Delta H^\circ}{R}\left(\frac{1}{T}\right)$

$y = c + mx$

Comparing with equation of straight line,

$\text{Slope} = -\frac{\Delta H^\circ}{R}$

Since, reaction is exothermic, ΔH° = –ve, therefore, slope = +ve.



Hence, option (3) is correct.

72. (3)

Assume the order of reaction with respect to acetaldehyde is x.

Condition-1:

$Rate = k[CH_3CHO]^x$

$1 = k[363 \times 0.95]^x$

$1 = k[344.85]^x$ .....(i)

Condition-2:

$0.5 = k[363 \times 0.67]^x$

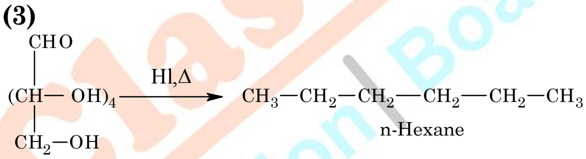
$0.5 = k[243.21]^x$ .....(ii)

Divide equation (i) by (ii),

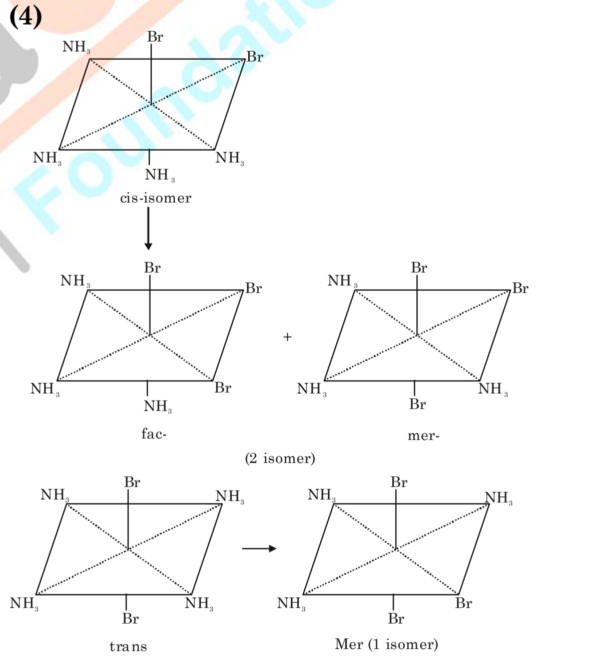
$\frac{1}{0.5} = \left(\frac{344.85}{243.21}\right)^x \Rightarrow 2 = (1.414)^x$

$\Rightarrow x = 2$

73. (3)



74. (4)

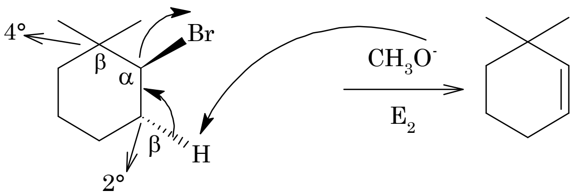


75. (4)

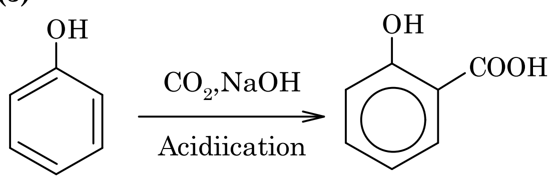
CH_3O^- is a strong base and strong nuclephile, so favourable condition is S_N2/E2.

Given alkyl halide is 2° and β C's are 4° and 2°, so sufficiently hindered, therefore, E2 dominates over S_N2.

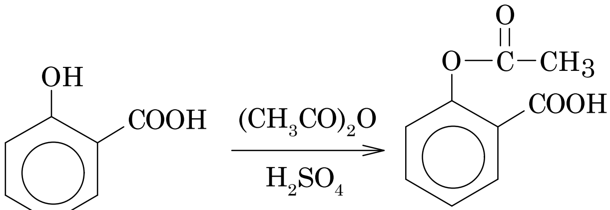
Also polarity of CH_3OH (solvent) is not as high as H_2O, so E1 is also dominated byE2



76. (3)



(Major)



Acetyl salicylic acid
Aspirin

77. (1)

Final concentration of $[SO_4^{--}] = \frac{[50 \times 1]}{500} = 0.1\ M$

K_{sp} of $BaSO_4$,

$[Ba^{2+}][SO_4^{2-}] = 1 \times 10^{-10}$

$[Ba^{2+}][0.1] = \frac{10^{-10}}{0.1} = 10^{-9}\ M$

Concentration of Ba^{2+} in final solution = 10^{-9} M

Concentration of Ba^{2+} in the original solution.

$M_1V_1 = M_2V_2$

$M_1\ (500 - 50) = 10^{-9}\ (500)$

$M1 = 1.11 \times 10^{-9}$ M

So, option (1) is correct.

78. (4)

Kjeldahl method is not applicable for compounds containing nitrogen in nitro, and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions. Hence only aniline can be used for estimation of nitrogen by Kjeldahl's method.

79. (1)

$$Al^{3+} \xrightarrow{NaOH} \underset{\text{White gelatinous ppt.}}{Al(OH)_3 \downarrow} \xrightarrow[NaOH]{Excess} \underset{\text{Sodium meta aluminate (soluble)}}{NaAlO_2}$$

$$2Al(OH)_3 \xrightarrow{\text{Strong heating}} Al_2O_3 + 3H_2O$$

Al_2O_3 is used in column chromatography.

80. (4)

In presence of external H^+

$H_2S \rightleftharpoons 2H^+ + S^{2-}$, $K_{a_1}.K_{a_2} = K_{eq}$

$\therefore \frac{[H^+]^2[S^{2-}]}{[H_2S]} = 1 \times 10^{-7} \times 1.2 \times 10^{-13}$

$\frac{[0.2]^2[S^{2-}]}{[0.1]} = 1.2 \times 10^{-20}$

$[S^{2-}] = 3 \times 10^{-20}$

81. (1)

F^- ions make the teeth enamel harder by converting

$\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]}$ and $\underset{\text{Fluorapatite}}{[3Ca_3(PO_4)_2.CaF_2]}$

82. (2)

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4H_2O + Cr_2O_3$

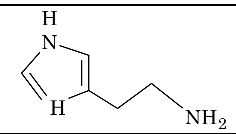
$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

$(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

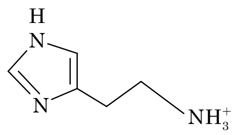
$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

Among all the given compounds, only $(NH_4)_2SO_4$ do not form dinitrogen on heating, it produces ammonia gas.

83. (2)

Histamine	

At pH (7.4) major form of histamine is protonated at primary amine.



84. (1)

$[Cr(H_2O)_6Cl_3 \Rightarrow x + 0 \times 6 - 1 \times 3 = 0$

$\therefore x = +3$

$[Cr(C_6H_6)_2 \Rightarrow x + 2 \times 0 = 0$

$x = 0$

$K_2[Cr(CN)_2(O_2)(O_2)NH_3]$

$\Rightarrow 1 \times 2 + x - 1 \times 2 - 2 \times 2 - 2 \times 1 = 0$

$\Rightarrow x - 6 = 0$

$x = +6$

85. (1)

In Frenkel defect, cation is dislocated from its normal lattice site to an interstitial site.

86. (2)

$C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H2O(l)$

$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$

$\Delta H = \Delta U + \Delta n_g RT$

$= -3263.9 + \left(-\frac{3}{2}\right) \times 8.314 \times 298 \times 10^{-3}$

$= -3263.9 + (-3.71)$

$= -3267.6 \text{ kJ mol}^{-1}$

87. (2)

BCl_3 – electron deficient, incomplete octet

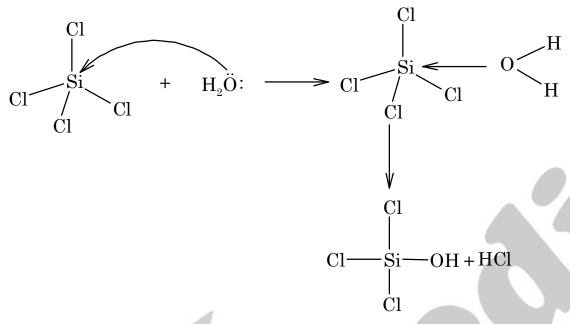
$AlCl_3$ – electron deficient, incomplete octet

Ans-(1) BCl_3 and $AlCl_3$

$SiCl_4$ can accept lone pair of electron in d-orbital of silicon hence it can act as Lewis acid.

* Although the most suitable answer is (1). However, both option (1) & (3) can be considered as correct answers.

e.g. hydrolysis of $SiCl_4$



Hence option (3), $AlCl_3$ and $SiCl_4$ is also correct.

88. (1)

KCl – Ionic bond between K^+ and Cl^-
PH_3 – Covalent bond between P and H
O_2 – Covalent bond between O atoms
B_2H_6 – Covalent bond between B and H atoms
H_2SO_4 – Covalent bond between S and O and also between O and H.
∴ Compound having no covalent bonds is KCl only.

89. (2)

The solution which shows maximum freezing point must have minimum number of solute particles.
(1) $[Co(H_2O)_3Cl_3] \cdot 3H_2O \rightarrow [Co(H_2O)_3Cl_3]$, i = 1
(2) $[Co(H_2O)_6]Cl_3 \rightarrow [Co(H_2O)_6]^{3+} + 3Cl^-$, i = 4
(3) $[Co(H_2O)_5Cl]Cl_2 \cdot 2H_2O \rightarrow [Co(H_2O)_5Cl]^{2+} + 2Cl^-$, i = 3
(4) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O \rightarrow [Co(H_2O)_4Cl_2]^{2+} + Cl^-$, i = 2
So, solution of 1 molal $[Co(H_2O_3)Cl_3] \cdot 3H_2O$ will have will have minimum number of particles in aqueous state.
Hence, option (1) is correct.

90. (2)

	Electronic configuration	Bond order
He_2^{2-}	$\sigma_{1s}^2 \sigma_{1s}^{*1}$	$\frac{2-1}{2} = 0.5$
H_2^-	$\sigma_{1s}^2 \sigma_{1s}^{*1}$	$\frac{2-1}{2} = 0.5$
He_2^{2-}	$\sigma_{1s}^2 \sigma_{1s}^{*2}$	$\frac{2-2}{2} = 0$
He_2^{2+}	σ_{1s}^2	$\frac{2-0}{2} = 1$

Molecular having zero bond order will not be a variable molecule.