MATHEMATICS

61. (c)

$$f(x)+2f\left(\frac{1}{x}\right)=3x \quad ......(i)$$

$$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x} \quad ....(ii)$$

On solving, $f(x) = \frac{2}{x}-x$

Now, $f(x) = f(-x)$

$\Rightarrow \frac{2}{x}-x = -\frac{2}{x}+x$

$\Rightarrow 2x-\frac{4}{x}=0$

$\Rightarrow x-\frac{2}{x}=0$

$\Rightarrow x^2-2=0$

$\Rightarrow x = \sqrt{2}, -\sqrt{2}$

62. (d)

$Re[(2+3i\sin\theta)(1+2i\sin\theta)] = 2-6\sin^2\theta = 0$

$\Rightarrow \sin^2\theta = \frac{1}{3}$

$Re[(2+3i\sin\theta)(1+2i\sin\theta)] = 2-6\sin^2\theta = 0$

63. (b)

$(x^2-5x+5)^{x^2+4x-60} = 1$

Case-I

$x^2 + 4x - 60 = 0$

$x = -10$

$x=6$

Case-II

$x^2 - 5x + 5 = 1$

$x^2 - 5x + 4 = 0$

$x = 1$

$x = 4$

Case-III

$x^2 - 5x + 5 = -1$

$x^2 - 5x + 6 = 0$

$x = 2$ or 3

For $x = 2$

$x^2 + 4x - 60 = -48$

For $x = 3$

$x^2 + 4x - 60 = -39$

$\therefore x = 2$

Sum of all real value = 3

64. (b)

$A = \begin{bmatrix} 5a & b \\ 3 & 2 \end{bmatrix}$

A adj. A = AA^T

$\Rightarrow \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix} = \begin{bmatrix} 25a+b & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$

$$\Rightarrow \begin{pmatrix} 15a - 2b = 0 & \Rightarrow b = 3,\ a = \frac{1}{5} \\ 10a + 3b = 13 & 5a + b = 10 = 13 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 15a - 2b = 0 & \Rightarrow b = 3,\ a = \frac{2}{5} \\ 10a + 3b = 13 & \Rightarrow 5a + b = 2 + 3 = 5 \end{pmatrix}$$

65. (d)

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - \lambda = 0$

$\Rightarrow \lambda(\lambda + 1)(\lambda - 1) = 0$

$\Rightarrow \lambda = 0, 1, -1$

66. (d)

Position of word 'SMALL' = 12 + 24 +12 +3 + 6 +1 = 58th

67. (d)

No. of terms = 28

$\Rightarrow \frac{1}{2}(n + 1)(n + 2) = 28$

$\Rightarrow n = 6$

sum of coefficients = 3^6 = 729

68. (b)

$a_2 = a + d$, $a_5 = a + 4d$, $a_9 = a + 8d$

$(a_5)^2 = a_2 \times a_9$

$(a + 4d)^2 = (a + d)(a + 8d)$

$a^2 + 16d^2 + 8ad = a^2 + 9ad + 8d^2$

$8d^2 = ad$

$a = 8d$

ratio $= \frac{a + 4d}{a + d} = \frac{8d + 4d}{8d + d} = \frac{12d}{9d} = \frac{4}{3}$

69. (b)

$$S_n = \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2$$

$$S_n = \frac{1}{25}[8^2 + 12^2 + 16^2 + 20^2 + ...]$$

$$S_n = \sum_{n=1}^{10} \frac{1}{25}\left[(4n + 4)^2\right]$$

$$= \sum_{n=1}^{10} \frac{16}{25}\left[n + 1^2\right]$$

$$= \sum_{n=1}^{10} \frac{16}{25}\left[n^2 + 2^n + 1\right] \quad 35.11$$

$$= \frac{16}{25}\left[\frac{10.11.21}{6} + \frac{10.11}{2} + 10\right]$$

$$= \frac{16}{25}\left[385 + 110 + 10\right]$$

$$= \frac{16}{25} \times 505 = \frac{16}{5} \times 101 \Rightarrow m = 101$$

70. (c)

$$P = \lim_{x \to \infty} + \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$$

$$= \lim_{e^{x \to 0}} + \frac{1}{2x} \times \tan^2 \sqrt{x}$$

$$= e^{\frac{1}{2}}$$

$$\Rightarrow \log p = \frac{1}{2}$$

71. (b)

$g(x) = f(f(x))$

$\Rightarrow g'(x) = f'(f(x))f'(x)$

$\Rightarrow g'(0) = f'(f(0))\, f'(0)$

For $x \to x, \log 2 > \sin x$

$\therefore f(x) = \log 2 - \sin x \quad \therefore f'(x) = -\cos x \Rightarrow f'(0) = -1$

Also, $x \to \log 2,\ \log 2 > \sin x \quad \therefore f(x) = \log 2 - \sin x$

$\therefore\ f'(x) = -\cos x \Rightarrow f'(\log 2) = -\cos(\log 2)$

$\therefore g'(0) = (-\cos(\log 2))\,(-1) = \cos(\log 2)$

72. (d)

$$f(x) = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$$

$$= \tan\left(\frac{\cos(x/2)+\sin(x/2)}{\cos(x/2)-\sin(x/2)}\right)$$

$$= \tan^{-1}\frac{1+\tan(x/2)}{1-\tan(x/2)}$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$= \frac{\pi}{4}+\frac{x}{2}$$

$$f'(x) = \frac{1}{2}$$

$\Rightarrow$ slope of normal $= -2$

at $x = \frac{\pi}{6},\ f\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$

$\therefore$ equation of normal is

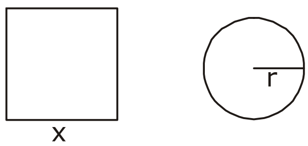
$$y - \frac{\pi}{3} = (-2)\left(x - \frac{\pi}{6}\right)$$

$$\Rightarrow y + 2x = \frac{2\pi}{3}$$

73. (c)

$4x + 2\pi r = 2$

$$\Rightarrow r = \frac{1-2x}{\pi}$$



$$A = \text{Area} = x^2 + \pi \times \frac{(1-2x)^2}{\pi 2} = x^2 + \frac{(1-2x)^2}{\pi}$$

$$\frac{dA}{dx} = 2x + \frac{2}{\pi}(1-2x)(-2) = 0$$

$$\Rightarrow x - \frac{2(1-2x)}{\pi} = 0$$

$$\Rightarrow x = \frac{2(1-2x)}{\pi} = \frac{2}{\pi} - \frac{4x}{\pi}$$

$$\Rightarrow \left(1 + \frac{4}{\pi}\right)x = \frac{2}{\pi}$$

$$\Rightarrow (\pi + 4)\pi = 2$$

$$\Rightarrow x = \frac{2}{\pi + 4} \Rightarrow \frac{x}{2} = \frac{1}{\pi + 4}$$

$$\therefore\ r = \frac{1-2x}{\pi} = \frac{1}{\pi} - \frac{2}{\pi} \times \frac{2}{\pi + 4} = \frac{1}{\pi} - \frac{4}{\pi(\pi + 4)}$$

$$= \frac{\pi + 4 - 4}{\pi(\pi + 4)} = \frac{1}{\pi + 4}$$

So, $\frac{x}{2} = r \Rightarrow x = 2r$

74. (b)

$$\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$$

$$= \int \frac{2x^{12} + 5x^9}{x^{15}(1 + x^{-2} + x^{-5})^3} dx$$

$$= \int \frac{2x^{-3} + 5x^{-6}}{(1 + x^{-2} + x^{-5})^3} dx$$

putting $1 + x^{-2} + x^{-6} = t$,
we get $dt = -(5x^{-6} + 2x^{-3})dx$

$$= -\int \frac{dt}{t^3} = -\left(\frac{t^{-2}}{-2}\right) = \frac{1}{2t^2} + c$$

$$= \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + c$$

75. (b)

$$\lim_{n\to\infty} \left\{\frac{(n+1)(n+2)......3n}{n^{2n}}\right\}^{1/n}$$

$$= \lim_{\pi\to\infty} \left\{\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n}\right)\left(\frac{n+2}{n}\right).......\right\}^{1/n}$$

$$\lim_{\pi\to\infty} \frac{1}{n}\left[\log\left(1 + \frac{1}{n}\right) + \log\left(1 + \frac{2}{n}\right) + ...\right]$$

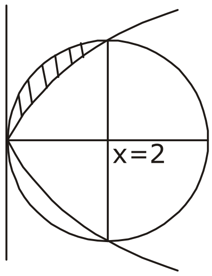
$$= e^{\int_0^2 \log(1+x)dx}$$

$$= e^{3\log 3 - 2}$$

$= e^{\log 27} \quad e^{-2}$

$= \frac{27}{e^2}$

76. (b)



Area $= \pi \times \frac{(2)^2}{4} - \sqrt{2}\int_0^2 \sqrt{x} \;\; dx$

$= \pi - \sqrt{2} \times \frac{2}{3}\left[x^{3/2}\right]_0^2$

$= \pi - \frac{2r_2}{3} \times 2\sqrt{2}$

$= \pi - \frac{8}{3}$

77. (d)

$\left[\frac{ydx - xdy}{d^2} + xdx = 0\right.$

$\frac{x}{y} + \frac{x^2}{2} = c$

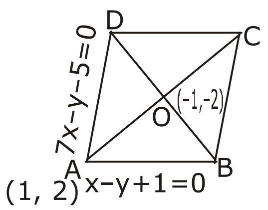
$x = 1, y = 1, -1 + \frac{1}{2} = c \Rightarrow c = -\frac{1}{2}$

so, $\frac{x}{y} + \frac{x^2}{2} = -\frac{1}{2}$

$x = -\frac{1}{2}, \; -\frac{1}{2y} + \frac{1}{8} = -\frac{1}{2}$

$\Rightarrow y = \frac{4}{5} \Rightarrow f\left(-\frac{1}{2}\right) = \frac{4}{5}$

78. (c)



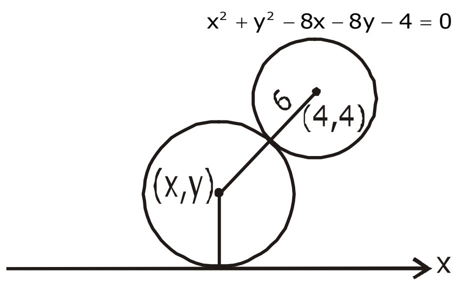
Equation of Diagonal BD is

$y + 2 = -\frac{1}{2}(x+1)$

$\Rightarrow x + 2y = -5$

going through options, one vertex is $\left(-\frac{1}{3}, -\frac{8}{3}\right)$

79. (d)

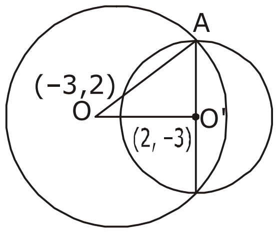


locus of centre of required circle is

$(x-4)^2 + (x-4)^2 = (x+6)^2$

$\Rightarrow x^2 - 8x - 20y - 4 = 0$ which is a parabola

80. (b)



$OO' = 5\sqrt{2}$, $O'A = 5$

$\Rightarrow OA = 5\sqrt{3}$

81. (a)

Let P $(2t^2, 4t)$

equation of normal $y + tx = 4t + 2t^3$

Normal must be passes through (0, −6)

$-6 + 0 = 4t + 2t^3$

$\Rightarrow t^3 + 2t + 3 = 0$

$\Rightarrow t = -1$

P = (2, −4), $r = \sqrt{4+4} = \sqrt{8}$

(x−2)2 + (y+4)2

$\Rightarrow x^2 - 4x + 4 + y^2 + 8y + 16 - 8 = 0$

$\Rightarrow x^2 + y^2 - 4x + 8y + 12 = 0$

82. (c)

$2b = C, \frac{2b2}{a} = 8$

$\Rightarrow b^2 = 4a$

$a^2 + b^2 = c^2 = 4b^2 = 16a$

$\Rightarrow a^2 + 4a = 16a$

$\Rightarrow a^2 = 12a \Rightarrow a = 12 \Rightarrow c^2 = 16\times12 \Rightarrow c = \sqrt{16\times12}$

$= 8\sqrt{3}$

$\Rightarrow e = \frac{8\sqrt{3}}{12} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$

83. (b)

Equation of line $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

Any point is $(\lambda+1, \lambda-5, \lambda+9)$

It lies on plane

$\Rightarrow (\lambda+1) - (\lambda-5) + (\lambda+9) = 5$

$\Rightarrow \lambda + 1 - \lambda + 5 + \lambda + 9 = 5$

$\Rightarrow \lambda = -10$

Point is (−9, −15, −1), another is (1, −5, 9)

Distance = $10\sqrt{3}$

84. (d)

$\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$, $\ell x + my - z = 9$

putting (3, −2, −4) , $3\ell - 2m + 4 = 9$

$\Rightarrow 3\ell - 2m = 5$ ..... (i)

Next $2 \times \ell + (-1)(m) + 3(1) = 0$

$\Rightarrow 2\ell - m = 3$ ......(ii)

solving (i) and (ii), $\ell = 1$, m=−1

$\ell^2 + m^2 = 2$

85. (d)

$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2} (\vec{b} + \vec{c})$, $\vec{b} \neq \vec{c}$

$\Rightarrow (\vec{a}.\ \vec{c})\ \vec{b} - (\vec{a}.\ \vec{b}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

$\Rightarrow \left(\vec{a}.\ \vec{c} - \frac{\sqrt{3}}{2}\right)\vec{b} - \left(\vec{a}.\ \vec{b} + \frac{\sqrt{3}}{2}\right)\vec{c} = 0$

$\Rightarrow \vec{a}.\ \vec{b} + \frac{\sqrt{3}}{2} = 0$

$\Rightarrow \vec{a}.\ \vec{b}. = -\frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$, $\Rightarrow \theta = \frac{5\pi}{6}$

86. (b)

Variance = $\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{4 + 9 + a^2 + 121}{4} - \left(4 + \frac{a}{4}\right)^2$

$= \frac{4(134 + a^2) - 256 - a^2 - 32a}{16}$

$3a^2 - 32a + 280 = 16\left(\frac{7}{2}\right)^2 = 4 \times 49$

$3a^2 - 32a + 84 = 0$

87. (d)

$P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{1}{6}$, $P(E_3) = \frac{1}{2}$

$P(E_1 \cap E_2) = \frac{1}{36}$, $P(E_2 \cap E_3)$, $P(E_2 \cap E_3) = \frac{1}{12}$, $P(E_1 \cap E_2) = \frac{1}{12}$

$P(E_1 \cap E_2 \cap E_3) = 0$

Clearly, E_1, E_2, E_3 are not independent.

88. (c)

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$\Rightarrow (\cos 4x + \cos x) + (\cos 3x + \cos 2x) = 0$

$\Rightarrow 2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2} \cdot \cos\frac{x}{2} = 0$

$\Rightarrow 2\cos\frac{5x}{2} \cdot \left(\cos\frac{3x}{2} + \cos\frac{x}{2}\right) = 0$

$\Rightarrow 2\cos\frac{5x}{2} \times 2\cos x.\ \cos\frac{x}{2} = 0$

$\Rightarrow \cos\frac{5x}{2} = 0,\ \cos x = 0,\ \cos\frac{x}{2} = 0$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos\frac{x}{2} = 0 \Rightarrow x = \pi$

$\cos\frac{5\pi}{2} = 0 \Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

89. (d)

Let speed = S m/sec
Distance = 600S

$\tan 60° = \frac{y}{x}$

$\sqrt{3} = \frac{y}{x}$

$y = \sqrt{3}x$

$\tan 30° = \frac{y}{x + 600S}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x + 600S}$

$x + 600S = 3x$
$600S = 2x$
$x = 300S$
Distance = 300S
Time = ?
Speed = S

Time = $\frac{300S}{S}$ = 300sec = 5 min.

90. (c)

$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$= [p \vee q) \wedge (\sim q \vee q)] \vee (\sim p \wedge q)$

$= [p \vee q) \wedge t] \vee (\sim p \wedge q)$

$= [p \vee q) \vee t] \vee (\sim p \wedge q)$

$= [(p \vee q \vee \sim p) \wedge (p \vee p \vee q)$

$= (t \vee q) \wedge (p \vee q) = t \wedge (p \vee q) = p \vee q$