MATHEMATICS

1. **(1)**

from $y^2 = 6x$, $\frac{dy}{dx} = \frac{3}{y}$

from $9x^2 + by^2 = 16$, $\frac{dy}{dx} = \frac{-ax}{by}$

ATQ, Curves cut at right angle,

$\frac{-ax}{by} \times \frac{3}{y} = -1 \Rightarrow by^2 = 27x \Rightarrow b = \frac{27x}{y^2} = \frac{27x}{6x} = \frac{9}{2}$

2. **(2)**

Let $\vec{u} = x\vec{a} + y\vec{b}$

Now $\vec{u} = \vec{a} \cdot \vec{b} = 0 \Rightarrow 14x + 2y = 0 \Rightarrow y = -7x$ ....(i)

$(\because |\vec{a}|^2 = 14, |\vec{b}|^2 = 2)$

$\vec{u} \cdot \vec{b} = 24 \Rightarrow 2x + 2y = 24$ $(\because \vec{a} \cdot \vec{b} = 2)$

$\Rightarrow x + y = 12$ ......(ii)

From (i) & (ii) $x = -2$, $y = 14$

$\therefore \vec{u} = -2(2\hat{i} + 3\hat{j} - \hat{k}) + 14(\hat{j} + \hat{k}) = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow |\vec{u}|^2 = 336$

3. **(4)**

Let $f(x) = x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

$= x\left(\frac{1}{x} + \frac{2}{x} + ... + \frac{15}{x} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right)\right)$

$= x\left(\frac{15 \times 16}{2x} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right)\right)$

$= 120 - x\left\{\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right\}$

We know $0 \le \left\{\frac{v}{x}\right\} < 1$

$\Rightarrow 0 \le x\left\{\frac{v}{x}\right\} < x$

$\Rightarrow \lim_{x \to 0^+} x\left\{\frac{v}{x}\right\} = 0$

$\Rightarrow \lim_{x \to 0} f(x) = 120$

4. **(3)**

Plane passing through $2x - 2y + 3z - 2 = 0$ any $x - y + z + 1 = 0$ is given by,

$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$

$\Rightarrow (\lambda + 2)x - (\lambda + 2)y + (\lambda + 3)z + (\lambda - 2) = 0$ ...(1)

Plane (1) and $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$ has infinitely many solution.

$$\Rightarrow \begin{vmatrix} \lambda + 2 & -(\lambda + 2) & \lambda + 3 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda = 5$

Equation of required plane is

$7x - 7y + 8z + 3 = 0$

Distance from origin $= \frac{3}{\sqrt{49 + 49 + 64}}$

$= \frac{3}{9\sqrt{2}} = \frac{1}{3\sqrt{2}}$

5. **(1)**

$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{2^x \sin^2 x}{1 + 2^x} dx$

$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = 2\int_0^{\pi/2} \sin^2 x dx$

$\Rightarrow I = \int_0^{\pi/2} \sin^2 dx = \int_0^{\pi/2} \cos^2 x\, dx$

$\Rightarrow 2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$

6. **(b)**

$18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow x = \frac{9\pi \pm 3\pi}{36} \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}$

i.e. $\alpha = \frac{\pi}{6}$, $\beta = \frac{\pi}{3}$

Area $= \int_{\pi/6}^{\pi/3} gof(x)\, dx$

$= \int_{\pi/6}^{\pi/3} \cos x\, dx = [\sin x]_{\pi/6}^{\pi/3} = \frac{1}{2}(\sqrt{3} - 1)$

7. (3)

$$8\cos x\left\{\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right\}=1$$

$$\Rightarrow \cos x\left\{\cos^2 x-\sin^2\frac{\pi}{6}-\frac{1}{2}\right\}=\frac{1}{8}$$

$$\Rightarrow \cos x\left(\cos^2 x-\frac{3}{4}\right)=\frac{1}{8}$$

$$\Rightarrow \frac{4\cos^3 x-3\cos x}{4}=\frac{1}{8}$$

$$\Rightarrow \cos 3x=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$\Rightarrow 3x=2n\pi\pm\frac{\pi}{3}\Rightarrow x=\frac{2n\pi}{3}\pm\frac{\pi}{9}$$

In $[0,\ \pi]$ sum of solutions $=\frac{\pi}{9}+\frac{2\pi}{3}+\frac{\pi}{9}+\frac{2\pi}{3}$

$K=\frac{13}{9}$ $\qquad\qquad =\frac{3\pi}{9}$

8. (1)

$$h(x)=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}=\frac{\left(x-\frac{1}{x}\right)^2+2}{x-\frac{1}{x}}$$

$$=\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}$$

We know, $\frac{\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}}{2}\ge\sqrt{\left(x-\frac{1}{x}\right)\times\frac{2}{\left(x-\frac{1}{x}\right)}}$

$$\Rightarrow \left(x-\frac{1}{x}\right)+\frac{2}{x-\frac{1}{x}}\ge 2\sqrt{2}$$

$\Rightarrow$ minimum value of h(x) $=2\sqrt{2}$

9. (3)

$$\int\frac{\sin^2 x\cos^2 x}{\left(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+1+\cos^5 x\right)^2}dx$$

$$=\int\frac{\sin^2 x\cos^2 x}{\cos^{10}x\left(\tan^5 x+\tan^2 x+\tan^3 x+1\right)}dx$$

$$=\int\frac{\frac{\sin^2 x}{\cos^2 x}\times\sec^6 x}{\left(\tan^3 x+1\right)^2\left(\tan^2 x+1\right)^2}dx$$

$$=\int\frac{\tan^2 x-\sec^6 x}{\left(\tan^3 x+1\right)^2\cdot\sec^4 x}dx$$

$$=\int\frac{\tan^2 x-\sec^2 x}{\left(1+\tan^3 x\right)^2}dx$$

Putting $1+\tan^3 x=t\ \Rightarrow\ dt=3\tan^2x\sec^2x\,dx$

$$=\frac{1}{3}\int\frac{1}{t^2}dt=-\frac{1}{3t}+c$$

$$=-\frac{1}{3\left(1+\tan^3 x\right)}+c$$

10. (3)

Required probability

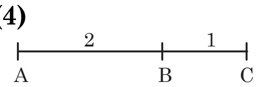
$$P(R_1)\,P\left(\frac{R_2}{R_1}\right)+P(B_1)\times P\left(\frac{R_2}{B_1}\right)$$

$$=\frac{4}{10}\times\frac{6}{12}+\frac{6}{10}\times\frac{4}{12}$$

$\left\{\begin{array}{l}R_1,R_2 \text{ are drawing red balls in Ist and 2nd draw}\\ B_1=\text{drawing black ball in Ist draw.}\end{array}\right.$

$$=\frac{2}{5}$$

11. (4)



Ortho centre   Centriode   (Circum centre)

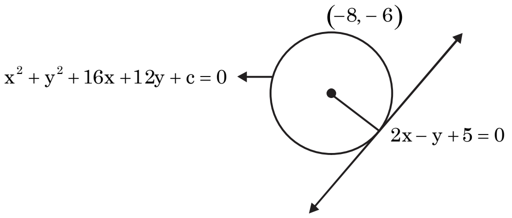
$$AC=\frac{3}{2}AB=\frac{3}{2}\times 2\sqrt{10}=3\sqrt{10}$$

Radius of circle with AC as diameter

$$=\frac{3}{2}\sqrt{10}\qquad=3\sqrt{\frac{5}{2}}$$

12. (1)

Tangent at (1, 7) to $x^2 = y - 6$ is $2x - y + 5 = 0$



Now, $\left|\dfrac{-16+6+5}{\sqrt{5}}\right| = \sqrt{64+36-C}$

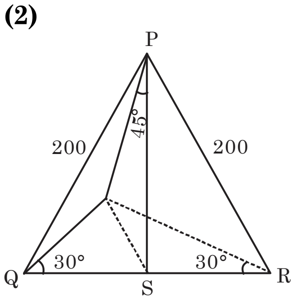
$\Rightarrow C = 95$

13. (4)

$\alpha = -\omega,\ \beta = -\omega^2$

$\therefore\ \alpha^{101} + \beta^{107} = -(\omega^2 + \omega) = 1$

14. (2)



Let height of tower

$ST = h$

In Δ QST, $\tan 30° = \dfrac{ST}{QS}$

$\Rightarrow QS = \sqrt{3}h = SR$

In Δ STP, ST = PS

In Δ PSQ, $PS = \sqrt{(200)^2 - (\sqrt{3}\,h)^2}$

So, $\sqrt{(200)^2 - 3h^2} = h \Rightarrow h = 100m$

15. (4)

$\sum_{i=0}^{9}(x_i - 5) = 9 \Rightarrow \sum_{i=1}^{9} x_i = 54$

Again $\sum_{i=1}^{9}(x_i = 5)^2 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 10\sum_{i=1}^{9} x_i + 225 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 360$

$$\text{Variance} = \sum_{i=1}^{9} x_i^2 - \left(\frac{\sum_{i=1}^{9} x_i}{9}\right)^2$$

$= \dfrac{360}{9} - \left(\dfrac{54}{9}\right)^2$

$= 4$

$\Rightarrow$ S.D = 2

16. (1)

Let $\sqrt{x^2 - 1} = a$

We have, $(x + a)^5 + (x - a)^5$

$= 2\left[{}^5C_0x^5 + {}^5C_2x^3a^2 + {}^5C_4x^5a^4\right]$

$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$

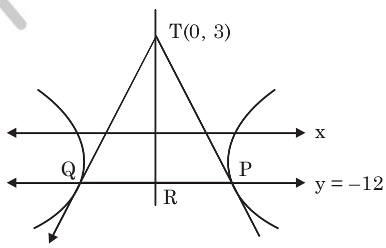
$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

Considering odd degree terms,

$2[x^5 + 5x^7 - 10x^3 + 5x]$

Sum of coefficients = 2

17. (2)



Area $= \dfrac{1}{2} \times PA \times TR$ $\quad \begin{cases} TR = 15 \\ PQ = 6\sqrt{5} \end{cases}$

$\dfrac{1}{2} \times 15 \times 6\sqrt{5}$

$= 45\sqrt{5}$

18. (2)

6 3

no. ways = $6 \times 6 \times 41$

4 1

= 1080

19. (3)

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0 \Rightarrow k = 11$$

$x + 11y + 3z = 0$ ______ (i)

$3x + 11y - 2z = 0$ ______ (ii)

$2x + 4y - 3z = 0$ ______ (iii)

+ (ii) $\Rightarrow x = -5y$

putting it in (i), we get

$-5y + 11y + 3z = 0$

$\Rightarrow z = -2y$

so, $\dfrac{xz}{y^2} = \dfrac{(-5y)(-2y)}{y^2} = 10$

20. (4)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (5x-4)(x+4)^2$$

so, eqating, $A = -4$, $B = 5$

21. (3)

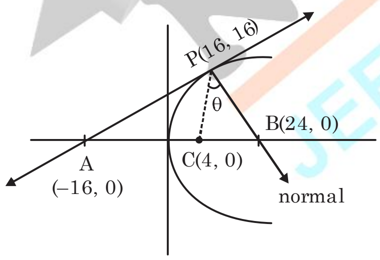
Set A contains all parts inside

$|x| < 1$ and $|y| < 1$

Set B contains all parts inside the ellipse all parts inside that ellipse $\dfrac{(x-1)^2}{9} + \dfrac{y^2}{4} = 1$

clearly ACB.

22. (3)



equation of tangent is
$x - 2y + 16 = 0$

$m_{PC} = \dfrac{4}{3}$

$m_{PB} = -2.$

$\therefore \tan\theta = \left|\dfrac{\frac{4}{3} + 2}{1 + \frac{4}{3} \times (-2)}\right| = 2$

23. (2)

$f(x) = |x - \pi| \left(e^{|x|} - 1\right) \sin|x|$

at $x = 0$

L.H.D = 0, R.H.d = 0

$\Rightarrow$ f(x) is differentiable.

$\therefore S = \phi$

24. (2)

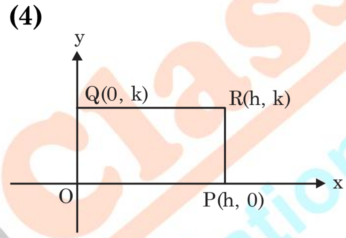
$\sim (p \vee g) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p \wedge (\sim q \vee q)$

$= \sim p \wedge t = \sim p$

25. (4)



Equation in PQ is

$\dfrac{x}{h} = \dfrac{y}{k} = 1$

Putting (2, 3), we get $\dfrac{2}{h} + \dfrac{3}{k} = 1$

$\therefore$ Locus will be,

$\dfrac{2}{x} + \dfrac{3}{y} = 1$

$\Rightarrow 3x + 2y = xy$

26. (3)

$B - 2A = \sum_{r=1}^{40} t_r - 2\sum_{r=1}^{20} t_r$

$(21^2 + 2.22^2 + ... + 40^2) - (1^2 + 2.2^2 + ... + 20^2)$

$= 20[(22 + 24 + ... + 60) + (24 + 28 + ... + 60)]$

$= 20\left[\dfrac{20}{2}(22 + 60) + \dfrac{10}{2}(24 + 60)\right]$

$100\lambda = 100 \times 248 \Rightarrow \lambda = 248$

27. (4)

$\sin x\, dy + y \cos x\, dx = 4x\, dx$

$\Rightarrow\ d(y \sin x) = 4x\, dx$

Integrating, $y \sin x = 2x^2 + c$

Curve passes through $\left(\frac{\pi}{2}, 0\right)$

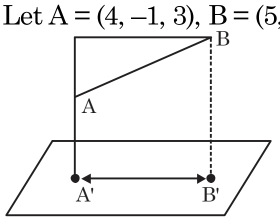
$\Rightarrow\ 0 = \frac{\pi^2}{2} + c \ \Rightarrow\ c = -\frac{\pi^2}{2}$

Now, $y \sin x = 2x^2 - \frac{\pi^2}{2}$

$\therefore\ y\left(\frac{\pi}{6}\right) = -\frac{8}{9}\pi^2$

28. (1)

Let A = (4, –1, 3), B = (5, –1, 4)



$AC = \overline{AB} \cdot \widehat{AC} = (\hat{i} + \hat{k}) \cdot \left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{\frac{2}{3}}$

$\Rightarrow$ Length of projection $= \sqrt{\frac{2}{3}}$

29. (1)

Case – 1 :

$2(3 - \sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow\ x - 8\sqrt{x} + 12 = 0 \ \Rightarrow\ \sqrt{x} = 4, 2$

$\Rightarrow$ x = 16 or x = 4 (Rejected)

Case – 2 :

Let $x \in [9, \infty)$

$2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow\ x - 4\sqrt{x} = 0 \ \Rightarrow$ x = 16 or x = 0 (Rejected)

So, x = 4, 16 are two solutions.

30. (4)

$\sum_{k=0}^{12} a_{4k+1} = 416$

$\Rightarrow\ \frac{13}{2}(2a_1 + 48d) = 416$

$\Rightarrow\ a_1 + 24d = 31$ ...(i)

$a_9 + a_{43} = 66 \Rightarrow\ 2a_1 + 50d = 66$ ...(ii)

from (i) and (ii), $d = 1$, $a_1 = 8$

Now, $140\, m = \sum_{r=1}^{17} ar^2$

$= \sum_{r=1}^{17} [8 - 1(r-1) \cdot 1]^2$

$= \sum_{r=1}^{17} (r+7)^2$

$= \sum_{r=1}^{24} r^2 - \sum_{r=1}^{7} r^2$

$= \frac{24 \times 25 \times 49}{6} - \frac{7 \times 8 \times 15}{6}$

$\Rightarrow\ m = 34$

PHYSICS

31. (3)

$\sigma = \frac{q_A}{4\pi a^2}$

$q_A = \sigma 4\pi a^2$

$q_b = -\sigma 4\pi b^2$

$q_c = \sigma 4\pi c^2$

$V_B = \frac{1}{4\pi \in 0} \sigma \left[\frac{4\pi a^2}{b} - \frac{q_B}{b} + \frac{q_C}{c}\right]$

$V_B = \frac{1}{4\pi \in 0} \sigma \left[\frac{4\pi a^2}{b} - \frac{4\pi b^2}{b} + \frac{4\pi c^2}{c}\right]$

$V_B = \frac{\sigma}{\in 0} \left[\frac{a^2 - b^2}{b} + c\right]$

32. (1)

$I' = \frac{MR^2}{2} + M(2R)^2$

$I' = \frac{MR^2}{2} + 4MR^2 = \frac{9}{2}MR^2$

$I = \frac{MR^2}{2} + 6 \times \frac{9}{2}MR^2$

$I = \frac{55}{2}MR^2$

$I_p = \frac{55}{2}MR^2 + 7M \times (3R)^2$

$I_p = \frac{181}{2}MR^2$

33. (2)

Mass of $\pi R^2 = 9M$

„ „ 1 Area $= \frac{9M}{\pi R^2}$

Mass of $\pi\left(\frac{R^2}{9}\right) = \frac{9M}{\pi R^2} \times \pi \frac{R^2}{9} = \frac{9M}{9} = M.$

Remaining moment of inertia

$I = I_{total} - I_{hole}$

$I = \frac{9MR2}{2} - \frac{MR^2}{2}$

$I_{hole} = \frac{M\left(\frac{R}{3}\right)^2}{2} + M\left(\frac{2R}{3}\right)^2$

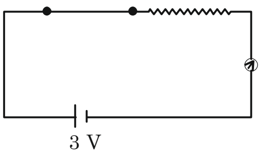
$I_{hole} = \frac{MR^2}{9\times 2} + \frac{4r^2MR^2}{9\times 2}$

$= \frac{5}{9}\frac{MR^2}{2}$

$I = 4MR^2$

34. (4)
Since the diode is non-ideal so there will be a voltage drop of 0.7V across it.

$i = \frac{3-0.7}{200}\times 10^3 = 11.5\text{ mA}$



35. (4)

$2^1 = \frac{2_0}{2}\cos^2\theta$

$\frac{2_0}{8} = \frac{2_0}{2}\cos^2\theta = \cos^2\theta$

$\frac{1}{4} = \cos^2\theta$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 45°$

36. (2)
Quality Factor

$Q = \frac{\omega_0 L}{R}$

37. (3)

$m_1 = 5$ kg
$m_2 = 10$ kg
$\mu = 0.15$

for stopping the motion.

$T = f_2 = \mu(m + m_2)g.$ ________ (1)

$T = m_1 g.$ ________(2)

$m_1 g = \mu(m_1 + m_2)g$

$m + m_2 = \frac{m_1}{\mu}$

$m = \frac{m_1}{\mu} - m_2 = \frac{5}{0.15} - 10 = \frac{500}{15} - 10$

$m = 23.33$ kg.

$\therefore$ Minimum $m = 27.3$ kg

38. (3)

$\frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 = \frac{150}{100}\times\frac{1}{2}mv_0^2$

$v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \rightarrow (1)$

$mv_0 = -mv_1 + mv_2$

$v_0 = v_2 - v_1 \rightarrow (2)$

Solving these 2 equations, we have

$v_1 + v_2 = \sqrt{2}\,v_0$

39. (4)

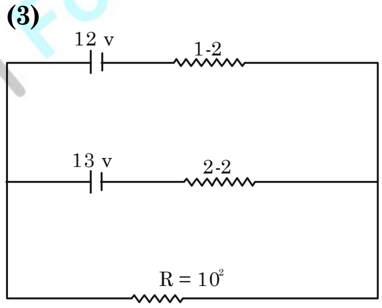
$F \propto \frac{1}{R^2}$

$\frac{mv^2}{R} = \frac{k}{R^n} \Rightarrow \frac{mv^2}{R^2} = m\omega^2 = \frac{k}{R^{n+1}}$

$\frac{m4\pi^2}{T^2} = \frac{k}{R^{n+1}}$

$\Rightarrow T \propto R^{(n+1)/2}$

40. (3)



$\frac{\sum\left(\frac{E}{r}\right)}{\sum\frac{1}{r}} = \text{Net Emf.}$

$\text{Net Emf} = \frac{\frac{12}{1}+\frac{13}{2}}{\left(\frac{1\times 2}{1\times 2}\right)\frac{3}{2}} = \frac{37}{3}$

$I = \frac{\left(\frac{37}{3}\right)}{\frac{2}{3}+10} = \frac{37}{3}\times\frac{3}{32} = \frac{37}{32}\text{A.}$

$V_R = IR$

$V_R = \frac{37}{32}\times 10 = 11.56\text{ volt}$

Max voltage = 11.56 which lies in only option (3).

41. (3)

$i_{rms} = \frac{20}{\sqrt{2}}$

walters current = $i_{rms} \sin\phi$

$= \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

= 10 A

power = $v_{rms}\, i_{rms} \cos\theta$

$\frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$

42. (4)

$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$

$\vec{E}_1 = E_{01}\hat{x}\cos\left[\left(\frac{2\pi c}{\lambda}\right)\left(\frac{z}{c} - t\right)\right]$

$\vec{E}_1 = E_{01}\hat{x}\cos[k(z - ct)]$

$k_1 = k$.

$\vec{E}_2 = E_{02}\hat{x}\cos\left[k2(z - \frac{c}{2}t)\right]$

$k_2 = 2k$

we know that

$k = \sqrt{\epsilon_r}$

$\frac{\epsilon_{r1}}{\epsilon_{r2}} = \left(\frac{k_1}{k_2}\right)$

$\frac{\epsilon_{r1}}{\epsilon_{r2}} = \left(\frac{k_1}{2k}\right)^2 = \frac{1}{4}$

43. (4)

10% of 10 6thr = 10 × 10 × $\frac{10}{100}$ = 109 Hz

No. of channels = $\frac{10^9}{5\times10^3} = \frac{10^6}{5} = 2\times10^5$

44. (2)

$v = \sqrt{\frac{y}{\rho}} = \sqrt{\frac{9.27\times10^{10}}{2.7\times10^3}}$

$v = \frac{1}{1.2}\sqrt{\frac{y}{\rho}}$

$v = \frac{1}{1.2}\sqrt{\frac{9.27\times10^{10}}{2.7\times10^3}}$

$v = 4.85\times10^3$

$v \approx 5$khz

45. (2)

$e = 1,\ v_2 = 0,\ u_1 = u$

$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u$

Fraction less of KE = $\frac{KEi - KE_f}{KE_i} = \frac{m_1v^2 - m_1v_1^2}{m_1v^2}$

$= \frac{v^2 - v_1^2}{v^2} = \frac{(m_1+m_2)^2 - (m_1-m_2)^2}{(m_1+m_2)^2} = \frac{4m_1m_2}{(m_1+m_2)^2}$

For deuterium, $m_1 = m$ and $m_2 = 2m$

$P_d = \frac{4\times m\times 2m}{(m+2m)^2} = \frac{8}{9} = 0.89$

For carbon, $m_1 = m$ and $m_2 = 12m$

$P_c = \frac{4\times m\times 12m}{(m+12m)^2} = \frac{48}{169} = 0.28$

46. (4)

$P = \frac{m}{V} = \frac{m}{\ell^3}$

$\frac{\Delta P}{P}\times100 = \frac{\Delta m}{m}\times100 + 3\frac{\Delta\ell}{\ell}\times100$

$= 1.5 + 3\times1$

$\frac{\Delta P}{P}\times100 = 4.5\%$

47. (4)

$n = 2,\ V_1 = V,\ T_1 = 27 + 273 = 300$ K

$\gamma = \frac{5}{3}$ (for monoatomic gas) $V_2 = 2V,\ T_2 = ?$

$TV^{\gamma-1}$ = Constant

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\left(\frac{V}{2V}\right)^{\frac{5}{3}-1}$

$T_2 = 300\left(\frac{1}{2}\right)^{\frac{2}{3}}$

$T_2 \approx 189$K

$dQ = dv + dw$

$dv = -dw$

$dv = -\frac{1}{\gamma-1}(P_1V_1 - P_2V_2)$

$dv = -\frac{nR}{\gamma-1}(300 - 189)$

$$dv = -\frac{2 \times 8.3}{0.6} \times 111 = -2.7 \text{ kJ}$$

48. (4)

$$K = \frac{\Delta P}{\Delta V / V}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta P}{K} = \frac{mg}{Ka}$$

$$\Rightarrow \frac{dr}{r} = \frac{1}{3}\frac{\Delta V}{V} = \frac{mg}{3Ka}$$

49. (2)

$q_0 = C_0V$

$q = KC_0V$

Induced charge, $q_i = q - q_0 = C_0V(K - 1)$

$= 90 \times 10^{-12} \times 20 \times 2/3$

$= 1200 \times 10^{-12} = 1.2$ nC

50. (4)

$M = iA = \pi r^2 A \Rightarrow M \propto r^2$

$$B = \frac{\mu_0 i}{2r} \Rightarrow B \propto \frac{1}{r}$$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{M_1}{M_2}}$$

Now, $\frac{M_1}{M_2} = \frac{r_1^2}{r_2^2} = \frac{1}{2}$

$$\Rightarrow \frac{r_1}{r_2} = \frac{B_2}{B_2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{B_1}{B_2} = \sqrt{2}$$

51. (2)

K.E. of $\bar{e}$ in n^{th} orbit $= \frac{13.6}{n^2} = \frac{1}{2}\frac{p^2}{m_e}$

$\Rightarrow P = \frac{\sqrt{27.2me}}{n} = \frac{k_1}{n}$ {Let say $K_1 = \sqrt{27.2me}$

Now $\lambda_n = \frac{h}{p} = \frac{hn}{k_1} \Rightarrow \lambda_n = k_2 n$

$$\frac{h}{\lambda n} = 13.6\left[1 - \frac{1}{n^2}\right] = 13.6$$

$$\frac{hc}{\wedge_n} = 13.6 - \frac{13.6}{n^2} = 13.6\left[\frac{n^2 - 1}{n^2}\right]$$

$$\wedge_n = \frac{hc}{13.6} \cdot \frac{n^2}{n^2 - 1} = \frac{hc}{13.6} + \frac{hc}{13.6(n^2 - 1)}$$

$$\wedge_n \approx \frac{hc}{13.6} + \frac{hc}{13.6n^2}$$

$$\Rightarrow \wedge_n = A + \frac{hcK_2^2}{13.6\lambda_n^2} = A + B/\lambda_n^2$$

52. (2)

$F = \Delta p / 1 \text{ sec} = 2mv\cos\theta \times n$

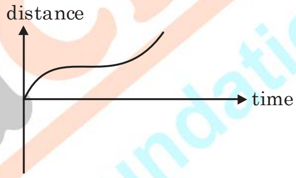
$$P = \frac{F}{A} = \frac{2mvn\cos\theta}{A} = \frac{2 \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23}}{2 \times 10^{-4} \times \sqrt{2}}$$

$= 2.35 \times 10^3$ N/m^2

53. (3)

If V vs time is a straight line with –ve slope, Acceleration = –ve (constant)

Then displacement vs time will be parabola like



Obiviously incorrect option is (3).

54. (3)

$$r = \frac{mv}{qB} = \frac{mv^2}{qBv} = \frac{2k}{qBv}$$

$$\Rightarrow r\alpha\frac{1}{qv} \Rightarrow r = \frac{k}{qv}$$

$m_\alpha = 4mp$

$m_e << mp$

As KE of all particles is equal

$V_e >> V_p$

$v_\alpha = {}^{v_p}/_2$

Let $q_e = 1$, $q_p = 1$, $q_\alpha = 2$

So $\quad re = \frac{k}{v_e}$

$r_p = \frac{k}{v_p}$

$$r_\alpha = \frac{k}{2 \times {}^{v_p}/_2} = \frac{k}{v_p}$$

So, $r_p = r_\alpha$ and $r_e < r_p$ [$\because$ $v_e >> v_p$]

$\therefore$ Correct order is $r_e < r_p = r_\alpha$

55. (4)

$$\frac{R_1}{\ell} = \frac{R_2}{100-\ell}$$

$$\frac{R_2}{\ell-10} = \frac{R_1}{110-\ell}$$

$$R_1 + R_2 = 1000$$

Solving these equations, we have

$R_1 = 550\,\Omega$ & $R_2 = 450\,\Omega$.

56. (3)

Let us assume voltage gradient of **wire** = K

Now emt of cell, e = 52k ____________ (i)

Terminal voltage of cell, v = 40k

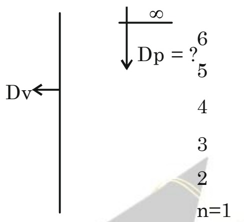
$$v = e - ir = e\left\{1 - \frac{r}{R+r}\right\} = 40\ k$$

$$\Rightarrow\ 52k\left[\frac{R}{R+r}\right] = 40\ k$$

$$\Rightarrow \frac{R}{R+r} = \frac{40}{52} = \frac{10}{13}$$

$$\Rightarrow \frac{5}{5+r} = \frac{10}{13} \Rightarrow r = 1.5\ \Omega$$

57. (1)



Lyman $\frac{1}{\lambda} = \frac{v}{c}$

$$\frac{1}{\lambda_v} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]200$$

$$\frac{v_1}{C} = R.C\left[\frac{1}{1}\right]$$

$RC = v_C$ ____________(i)

Phund

$$\frac{v_P}{C} = \frac{1}{\lambda_p} = R\left[\frac{1}{(5)^2} - \frac{1}{\infty}\right]$$

$$v_P = \frac{v_L}{25}$$

58. (2)

$2\theta = 60°$

$\theta = 30°$

$d\ \sin\theta = \lambda$, (d=width of single slit=1 μ m = 10^{-6} m.)

$\lambda = 10^{-6}$ Sin30°

$\lambda = 0.5 \times 10^{-6}$

for Double Slit

$$\beta = \frac{D\lambda}{d} \quad \text{(d = slit separation, D= 50 cm = 0.5 m)}$$

$$10^{-2} = \frac{0.5 \times 0.5 \times 10^{-6}}{d}$$

$d = 25 \times 10^{-6}$ m.

$d = 25\ \mu$ m

59. (4)

$$U = \frac{-k}{2r^2}$$

$$F = \frac{-dU}{dr} = +\frac{d}{dr}\left(\frac{k}{2r^2}\right) = \frac{k}{2}\frac{d}{dr}\left(r^{-2}\right)$$

$$F = \frac{k}{2}(-2)r^{-3} = \frac{-k}{r^3} = \frac{mv^2}{r}$$

$$K.E = \left|\frac{1}{2}mv^2\right| = \left|\frac{-k}{2r^2}\right| = \frac{k}{2r^2}$$

T.E = K.E + P.E

$$T.E = \frac{k}{2r^2} - \frac{k}{2r^2} = 0$$

60. (3)

$\nu = 10^{12}$ Hz

$108 = 6.02 \times 10^{23}$ m.

$$m = \frac{108 \times 10^{-3}}{6.02 \times 10^{23}} kg$$

$$\frac{1}{\nu} = T = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{1}{\nu^2} = 4\pi^2\frac{m}{k}$$

$$k = 4\pi^2 m\nu^2$$

$$k = \frac{4 \times 9.8596 \times 108 \times 10^{+24}}{6.02 \times 10^{23}}$$

k = 7.07 N/m.

CHEMISTRY

61. (1)

The solution which shows maximum freezing point must have minimum number of solute particles.

(1) $[Co(H_2O)_3Cl_3] \cdot 3H_2O \rightarrow [Co(H_2O)_3Cl_3]$, i = 1

(2) $[Co(H_2O)_6]Cl_3 \rightarrow [Co(H_2O)_6]^{3+} + 3Cl^-$, i = 4

(3) $[Co(H_2O)_5Cl]Cl_2 \cdot 2H_2O \rightarrow [Co(H_2O)_5Cl]^{2+} + 2Cl^-$, i = 3

(4) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O \rightarrow [Co(H_2O)_4Cl_2]^{2+} + Cl^-$, i = 2

So, solution of 1 molal $[Co(H_2O_3)Cl_3] \cdot 3H_2O$ will have will have minimum number of particles in aqueous state.

Hence, option (1) is correct.

62. (4)

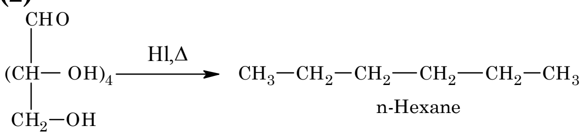
$$[Fe(CN)_6]^{4-} + \frac{1}{2} H_2O_2 + H^+ \rightarrow [Fe(CN)_6]^{3-} + H_2O$$

$$[Fe(CN)_6]^{3-} + \frac{1}{2} H_2O_2 + OH^{\ominus} \rightarrow$$

$$[Fe(CN_6)^{4-}] + H_2O + \frac{1}{2} O_2$$

63. (3)
Kjeldahl method is not applicable for compounds containing nitrogen in nitro, and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions. Hence only aniline can be used for estimation of nitrogen by Kjeldahl's method.

64. (2)

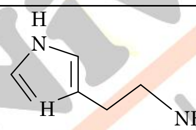


65. (4)

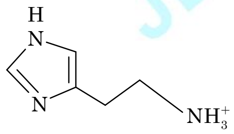
$\longleftarrow$ 3.9 — 4.5 $\longrightarrow$
Pinkish red Yellow

Weak base is having pH greater than 7. When methyl orange is added to weak base solution, the solution becomes yellow. This solution is titrated by strong acid and at the end point pH will be less than 3.1. Therefore solution becomes pinkish red.

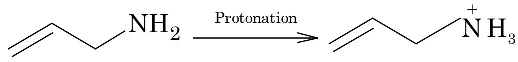
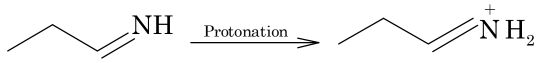
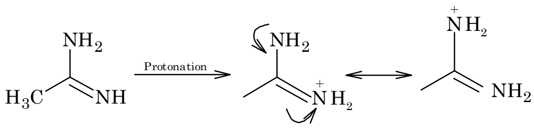
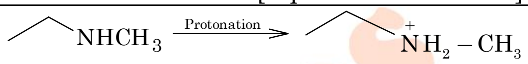
66. (1)

Histamine	

At pH (7.4) major form of histamine is protonated at primary amine.



67. (4)

(a)	 1° & sp^3
(b)	 sp^2
(c)	 [Equivalent resonance]
(d)	 2° & sp^3

$\therefore$ Correct order of basically : b < a < d < c.

68. (2)

Equilibrium constant $K = \left(\frac{A_t}{A_b}\right) e^{-\frac{\Delta H^\circ}{RT}}$

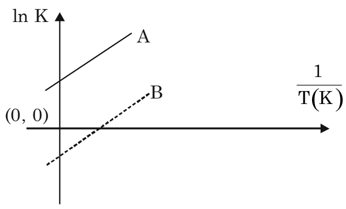
$$\ln K = \ln\left(\frac{A_t}{A_b}\right) - \frac{\Delta H^\circ}{R}\left(\frac{1}{T}\right)$$

$y = c + mx$

Comparing with equation of straight line,

$$\text{Slope} = -\frac{\Delta H^\circ}{R}$$

Since, reaction is exothermic, ΔH° = –ve, therefore, slope = +ve.



Hence, option (2) is correct.

69. (4)

$B_2H_6 + 3O_2 \rightarrow B_2O_3 + 3H_2O$

27.66 of B_2H_6 = 1 mole of B_2H_6 which requires three moles of oxygen (O_2) for complete burning

$6H_2O \rightarrow 6H_2 + 3O_2$ (On electrolysis)

Number of faradays = 12 = Amount of charge

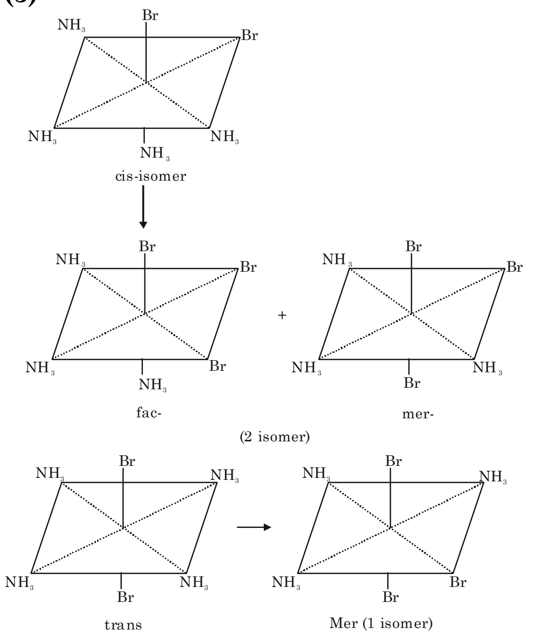
$12 \times 96500 = i \times t$

$12 \times 96500 = 100 \times t$

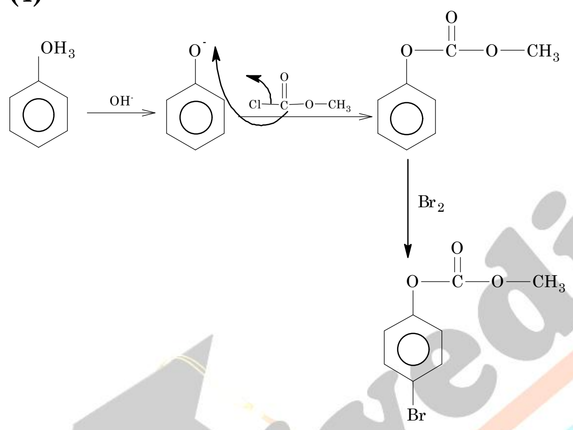
$$t = \frac{12 \times 96500}{100} \text{ second}$$

$$t = \frac{12 \times 96500}{100 \times 3600} \text{ hour} \Rightarrow t = 3.2 \text{ hours}$$

70. (3)



71. (4)



72. (4)

Final concentration of $[SO_4^{--}] = \frac{[50 \times 1]}{500} = 0.1\text{ M}$

K_{sp} of $BaSO_4$,

$[Ba^{2+}][SO_4^{2-}] = 1 \times 10^{-10}$

$[Ba^{2+}][0.1] = \frac{10^{-10}}{0.1} = 10^{-9}\text{ M}$

Concentration of Ba^{2+} in final solution = 10^{-9} M

Concentration of Ba^{2+} in the original solution.

$M_1V_1 = M_2V_2$

$M_1 (500 - 50) = 10^{-9} (500)$

$M1 = 1.11 \times 10^{-9}$ M

So, option (4) is correct.

73. (2)

Assume the order of reaction with respect to acetaldehyde is x.

Condition-1:

Rate = $k[CH_3CHO]^x$

$1 = k[363 \times 0.95]^x$

$1 = k[344.85]^x$ .....(i)

Condition-2:

$0.5 = k[363 \times 0.67]^x$

$0.5 = k[243.21]^x$ .....(ii)

Divide equation (i) by (ii),

$$\frac{1}{0.5} = \left(\frac{344.85}{243.21}\right)^x \Rightarrow 2 = (1.414)^x$$

$\Rightarrow$ x = 2

74. (1)

$$C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H2O(l)$$

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 + \left(-\frac{3}{2}\right) \times 8.314 \times 298 \times 10^{-3}$$

$$= -3263.9 + (-3.71)$$

$$= -3267.6 \text{ kJ mol}^{-1}$$

75. (1)

Element	Relative mass	Relative mole	Simplest whole number ratio
C	6	$\frac{6}{12} = 0.5$	1
H	1	$\frac{1}{1} = 1$	2

So, X = 1, Y = 2

Equation for combustion of C_XH_Y

$$C_XH_Y + \left(X + \frac{Y}{4}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

Oxygen atoms required $= 2\left(X + \frac{Y}{4}\right)$

As per information,

$$2\left(X+\frac{Y}{4}\right)=2Z$$

$$\Rightarrow \left(1+\frac{2}{4}\right)=Z$$

$\Rightarrow Z = 1.5$

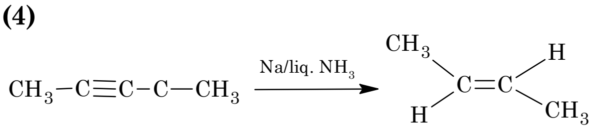
Molecule can be written

$C_XH_YO_Z$

$C_1H_2O_{3/2}$

$\Rightarrow C_2H_4O_3$

76. (4)



So, option (4) is correct.

77. (1)

BCl_3 – electron deficient, incomplete octet

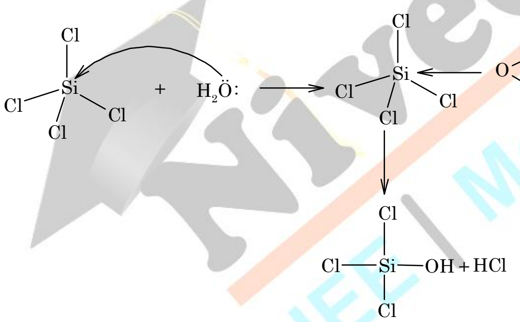
$AlCl_3$ – electron deficient, incomplete octet

Ans-(1) BCl_3 and $AlCl_3$

$SiCl_4$ can accept lone pair of electron in d-orbital of silicon hence it can act as Lewis acid.

* Although the most suitable answer is (1). However, both option (1) & (3) can be considered as correct answers.

e.g. hydrolysis of $SiCl_4$



Hence option (3), $AlCl_3$ and $SiCl_4$ is also correct.

78. (4)

$Al^{3+} \xrightarrow{NaOH} \underset{\text{White gelatinous ppt.}}{Al(OH)_3\downarrow} \xrightarrow[NaOH]{Excess} \underset{\text{Sodium meta aluminate (soluble)}}{NaAlO_2}$

$2Al(OH)_3 \xrightarrow{\text{Strong heating}} Al_3O_3 + 3H_2O$

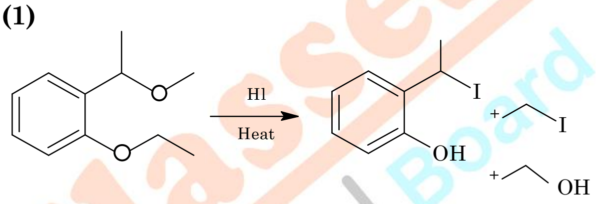
Al_2O_3 is used in column chromatography.

79. (1)

	Electronic configuration	Bond order
He_2^{2-}	$\sigma_{1s^2}\sigma^*_{1s^1}$	$\frac{2-1}{2}=0.5$
H_2^-	$\sigma_{1s^2}\sigma^*_{1s^1}$	$\frac{2-1}{2}=0.5$
He_2^{2-}	$\sigma_{1s^2}\sigma^*_{1s^2}$	$\frac{2-2}{2}=0$
He_2^{2+}	σ_{1s^2}	$\frac{2-0}{2}=1$

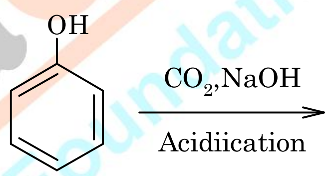
Molecular having zero bond order will not be a variable molecule.

80. (1)

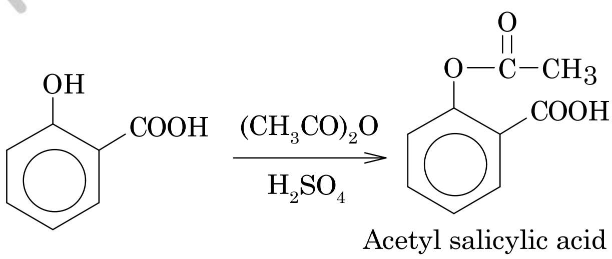


Hence, option (1) is correct.

81. (2)



(Major)



Acetyl salicylic acid
Aspirin

82. (4)

KCl – Ionic bond between K^+ and Cl^-

PH_3 – Covalent bond between P and H

O_2 – Covalent bond between O atoms

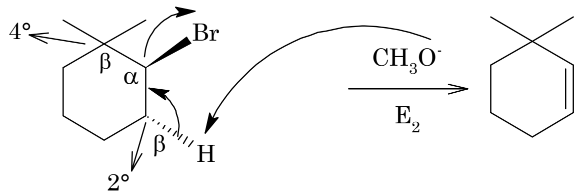
B_2H_6 – Covalent bond between B and H atoms

H_2SO_4 – Covalent bond between S and O and also between O and H.

∴ Compound having no covalent bonds is KCl only.

83. (4)
In Frenkel defect, cation is dislocated from its normal lattice site to an interstitial site.

84. (3)
CH_3O^- is a strong base and strong nuclephile, so favourable condition is $S_N2/E2$.
Given alkyl halide is 2° and β C's are 4° and 2°, so sufficiently hindered, therefore, E2 dominates over S_N2.
Also polarity of CH_3OH (solvent) is not as high as H_2O, so E1 is also dominated byE2



85. (1)
$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4H_2O + Cr_2O_3$
$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$
$(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$
$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$
Among all the given compounds, only $(NH_4)_2SO_4$ do not form dinitrogen on heating, it produces ammonia gas.

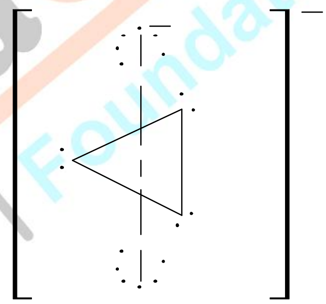
86. (3)
In presence of external H^+
$H_2S \rightleftharpoons 2H^+ + S^{2-}, K_{a_1}.K_{a_2} = K_{eq}$
$\therefore \frac{[H^+]^2[S^{2-}]}{[H_2S]} = 1\times10^{-7} \times 1.2 \times 10^{-13}$
$\frac{[0.2]^2[S^{2-}]}{[0.1]} = 1.2\times10^{-20}$
$[S^{2-}] = 3 \times 10^{-20}$

87. (4)
$[Cr(H_2O)_6Cl_3 \Rightarrow x + 0 \times 6 - 1 \times 3 = 0$
$\therefore x = +3$
$[Cr(C_6H_6)_2 \Rightarrow x + 2 \times 0 = 0$
$x = 0$
$K_2[Cr(CN)_2(O_2)(O_2)NH_3]$
$\Rightarrow 1 \times 2 + x - 1 \times 2 - 2 \times 2 - 2 \times 1 = 0$
$\Rightarrow x - 6 = 0$
$x = +6$

88. (4)
F^- ions make the teeth enamel harder by converting
$\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]}$ and $\underset{\text{Fluorapatite}}{[3Ca_3(PO_4)_2.CaF_2]}$

89. (3)
$CH_3COOK + H_2O \rightarrow CH_3COOH + KOH$ Basic
$FeCl_3$ – Acidic solution
$Al(CN)_3$ – Salt of weak acid and weak base
$Pb(CH_3COO)_2$ – Salt of weak acid and weak base
CH_3COOK is salt of weak acid and strong base.
Hence solution of CH_3COOK is basic.

90. (4)
Structure of I_3^-



Number of lone pairs in $I_3^{\ominus}$ is 9.