JEE MAIN MATH SOLUTIONS – CODE D
Held on : 08 April 2018

MATHEMATICS

1. **(1)**

from $y^2 = 6x$, $\frac{dy}{dx} = \frac{3}{y}$

from $9x^2 + by^2 = 16$, $\frac{dy}{dx} = \frac{-ax}{by}$

ATQ, Curves cut at right angle,

$\frac{-ax}{by} \times \frac{3}{y} = -1 \Rightarrow by^2 = 27x \Rightarrow b = \frac{27x}{y^2} = \frac{27x}{6x} = \frac{9}{2}$

2. **(2)**
Let $\vec{u} = x\vec{a} + y\vec{b}$

Now $\vec{u} = \vec{a}\cdot\vec{b} = 0 \Rightarrow 14x + 2y = 0 \Rightarrow y = -7x$ ....(i)

$(\because |\vec{a}|^2 = 14, |\vec{b}|^2 = 2)$

$\vec{u}\cdot\vec{b} = 24 \Rightarrow 2x + 2y = 24$ $\quad (\because \vec{a}\cdot\vec{b} = 2)$

$\Rightarrow x + y = 12$ ......(ii)

From (i) & (ii) $x = -2$, $y = 14$

$\therefore \vec{u} = -2(2\hat{i} + 3\hat{j} - \hat{k}) + 14(\hat{j} + \hat{k}) = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow |\vec{u}|^2 = 336$

3. **(4)**

Let $f(x) = x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots\ldots + \left[\frac{15}{x}\right]\right)$

$= x\left(\frac{1}{x} + \frac{2}{x} + \ldots\ldots + \frac{15}{x} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots\ldots + \left\{\frac{15}{x}\right\}\right)\right)$

$= x\left(\frac{15\times 16}{2x} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots\ldots + \left\{\frac{15}{x}\right\}\right)\right)$

$= 120 - x\left\{\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots\ldots + \left\{\frac{15}{x}\right\}\right\}$

We know $0 \le \left\{\frac{v}{x}\right\} < 1$

$\Rightarrow 0 \le x\left\{\frac{v}{x}\right\} < x$

$\Rightarrow \lim_{x\to 0^+} x\left\{\frac{v}{x}\right\} = 0$

$\Rightarrow \lim_{x\to 0} f(x) = 120$

4. **(3)**
Plane passing through $2x - 2y + 3z - 2 = 0$ any $x - y + z + 1 = 0$ is given by,

$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$

$\Rightarrow (\lambda + 2)x - (\lambda + 2)y + (\lambda + 3)z + (\lambda - 2) = 0$ ...(1)

Plane (1) and $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$ has infinitely many solution.

$$\Rightarrow \begin{vmatrix} \lambda + 2 & -(\lambda + 2) & \lambda + 3 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda = 5$

Equation of required plane is

$$7x - 7y + 8z + 3 = 0$$

Distance from origin $= \frac{3}{\sqrt{49 + 49 + 64}}$

$= \frac{3}{9\sqrt{2}} = \frac{1}{3\sqrt{2}}$

5. **(1)**

$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{2^x \sin^2 x}{1 + 2^x} dx$

$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = 2\int_0^{\pi/2} \sin^2 x dx$

$\Rightarrow I = \int_0^{\pi/2} \sin^2 dx = \int_0^{\pi/2} \cos^2 x\, dx$

$\Rightarrow 2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$

6. **(b)**
$18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow x = \frac{9\pi \pm 3\pi}{36} \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{3}$

i.e. $\alpha = \frac{\pi}{6}$, $\beta = \frac{\pi}{3}$

Area $= \int_{\pi/6}^{\pi/3} gof(x)\, dx$

$= \int_{\pi/6}^{\pi/3} \cos x\, dx = [\sin x]_{\pi/6}^{\pi/3} = \frac{1}{2}\left(\sqrt{3} - 1\right)$

7. **(3)**

$8\cos x\left\{\cos\left(\frac{\pi}{6} + x\right)\cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right\} = 1$

$\Rightarrow \cos x\left\{\cos^2 x - \sin^2\frac{\pi}{6} - \frac{1}{2}\right\} = \frac{1}{8}$

$\Rightarrow \cos x\left(\cos^2 x - \frac{3}{4}\right) = \frac{1}{8}$

$\Rightarrow \frac{4\cos^3 x - 3\cos x}{4} = \frac{1}{8}$

$\Rightarrow \cos 3x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow 3x = 2n\pi \pm \frac{\pi}{3} \Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

In $[0, \pi]$ sum of solutions $= \frac{\pi}{9} + \frac{2\pi}{3} + \frac{\pi}{9} + \frac{2\pi}{3}$

$K = \frac{13}{9}$ $\qquad = \frac{3\pi}{9}$

8. **(1)**

$$h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \frac{\left(x - \frac{1}{x}\right)^2 + 2}{x - \frac{1}{x}}$$

$$= \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}$$

We know, $\frac{\left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}}{2} \ge \sqrt{\left(x - \frac{1}{x}\right) \times \frac{2}{\left(x - \frac{1}{x}\right)}}$

$\Rightarrow \left(x - \frac{1}{x}\right) + \frac{2}{x - \frac{1}{x}} \ge 2\sqrt{2}$

$\Rightarrow$ minimum value of $h(x) = 2\sqrt{2}$

9. **(3)**

$$\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + 1 + \cos^5 x\right)^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{\cos^{10} x \left(\tan^5 x + \tan^2 x + \tan^3 x + 1\right)} dx$$

$$= \int \frac{\frac{\sin^2 x}{\cos^2 x} \times \sec^6 x}{\left(\tan^3 x + 1\right)^2 \left(\tan^2 x + 1\right)^2} dx$$

$$= \int \frac{\tan^2 x - \sec^6 x}{\left(\tan^3 x + 1\right)^2 \cdot \sec^4 x} dx$$

$$= \int \frac{\tan^2 x - \sec^2 x}{\left(1 + \tan^3 x\right)^2} dx$$

Putting $1 + \tan^3 x = t \;\Rightarrow\; dt = 3\tan^2 x \sec^2 x \, dx$

$$= \frac{1}{3}\int \frac{1}{t^2} dt = -\frac{1}{3t} + c$$

$$= -\frac{1}{3\left(1 + \tan^3 x\right)} + c$$

10. **(3)**

Required probability

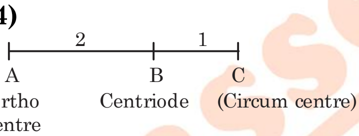
$P(R_1)\, P\left(\frac{R_2}{R_1}\right) + P(B_1) \times P\left(\frac{R_2}{B_1}\right)$

$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12}$

$\left\{ \begin{array}{l} R_1, R_2 \text{ are drawing red balls} \\ \text{in Ist and 2nd draw} \\ B_1 = \text{drawing black ball} \\ \quad \text{in Ist draw.} \end{array} \right.$

$= \frac{2}{5}$

11. **(4)**



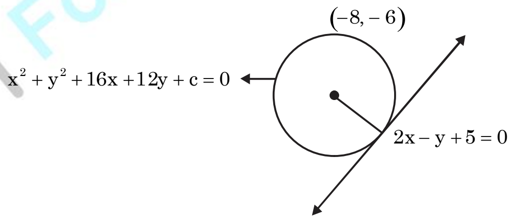
$AC = \frac{3}{2} AB = \frac{3}{2} \times 2\sqrt{10} = 3\sqrt{10}$

Radius of circle with AC as diameter

$= \frac{3}{2}\sqrt{10} \quad = 3\sqrt{\frac{5}{2}}$

12. **(1)**

Tangent at (1, 7) to $x^2 = y - 6$ is $2x - y + 5 = 0$



Now, $\left|\frac{-16 + 6 + 5}{\sqrt{5}}\right| = \sqrt{64 + 36 - C}$

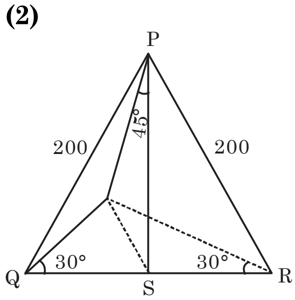
$\Rightarrow C = 95$

13. **(4)**

$\alpha = -\omega, \ \beta = -\omega^2$

$\therefore \ \alpha^{101} + \beta^{107} = -(\omega^2 + \omega) = 1$

14. **(2)**



Let height of tower

ST = h

In Δ QST, $\tan 30° = \dfrac{ST}{QS}$

$\Rightarrow$ QS = $\sqrt{3}h$ = SR

In Δ STP, ST = PS

In Δ PSQ, PS = $\sqrt{(200)^2 - (\sqrt{3}\,h)^2}$

So, $\sqrt{(200)^2 - 3h^2} = h \Rightarrow h = 100\text{m}$

15. **(4)**

$\sum_{i=0}^{9}(x_i - 5) = 9 \;\Rightarrow\; \sum_{i=1}^{9} x_i = 54$

Again $\sum_{i=1}^{9}(x_i = 5)^2 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 10\sum_{i=1}^{9} x_i + 225 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 360$

Variance = $\sum_{i=1}^{9} x_i^2 - \left(\dfrac{\sum_{i=1}^{9} x_i}{9}\right)^2$

$= \dfrac{360}{9} - \left(\dfrac{54}{9}\right)^2$

= 4

$\Rightarrow$ S.D = 2

16. **(1)**

Let $\sqrt{x^2 - 1} = a$

We have, $(x + a)^5 + (x - a)^5$

$= 2\left[{}^5C_0x^5 + {}^5C_2x^3a^2 + {}^5C_4x^5a^4\right]$

$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$

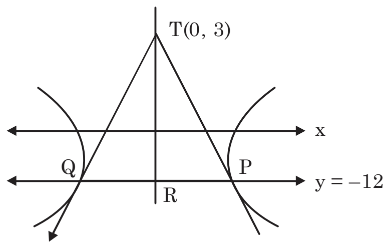
$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

Considering odd degree terms,

$2[x^5 + 5x^7 - 10x^3 + 5x]$

Sum of coefficients = 2

17. **(2)**



Area = $\dfrac{1}{2} \times PA \times TR$ $\begin{cases} TR = 15 \\ PQ = 6\sqrt{5} \end{cases}$

$\dfrac{1}{2} \times 15 \times 6\sqrt{5}$

$= 45\sqrt{5}$

18. **(2)**

no. ways = $\begin{matrix}6 & 3 \\ 6\times 6\times 41 \\ 4 & 1\end{matrix}$

= 1080

19. **(3)**

$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0 \Rightarrow k = 11$

$x + 11y + 3z = 0$ _______ (i)

$3x + 11y - 2z = 0$ _______ (ii)

$2x + 4y - 3z = 0$ _______ (iii)

(i) + (ii) $\Rightarrow x = -5y$

putting it in (i), we get

$-5y + 11y + 3z = 0$

$\Rightarrow z = -2y$

so, $\dfrac{xz}{y^2} = \dfrac{(-5y)(-2y)}{y^2} = 10$

20. **(4)**

$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (5x-4)(x+4)^2$

so, eqating, A = –4, B = 5

21. **(3)**

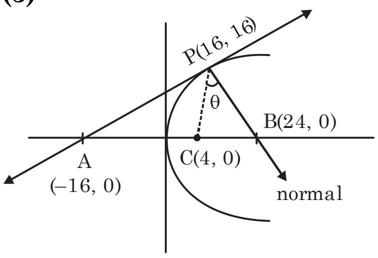
Set A contains all parts inside

$|x| < 1$ and $|y| < 1$

Set B contains all parts inside the ellipse all parts inside that ellipse $\dfrac{(x-1)^2}{9} + \dfrac{y^2}{4} = 1$

clearly ACB.

22. (3)



equation of tangent is
$x - 2y + 16 = 0$

$m_{PC} = \frac{4}{3}$

$m_{PB} = -2.$

$\therefore \tan\theta = \left|\frac{\frac{4}{3}+2}{1+\frac{4}{3}\times(-2)}\right| = 2$

23. (2)

$f(x) = |x-\pi|\left(e^{|x|}-1\right)\sin|x|$

at $x = 0$

L.H.D = 0, R.H.d = 0

$\Rightarrow$ f(x) is differentiable.

$\therefore\ S = \phi$

24. (2)

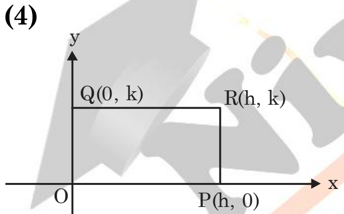
$\sim(p \vee g) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p \wedge (\sim q \vee q)$

$= \sim p \wedge t = \sim p$

25. (4)



Equation in PQ is

$\frac{x}{h} = \frac{y}{k} = 1$

Putting (2, 3), we get $\frac{2}{h} + \frac{3}{k} = 1$

$\therefore$ Locus will be,

$\frac{2}{x} + \frac{3}{y} = 1$

$\Rightarrow 3x + 2y = xy$

26. (3)

$B - 2A = \sum_{r=1}^{40} t_r - 2\sum_{r=1}^{20} t_r$

$(21^2 + 2.22^2 + \ldots\ldots + 40^2) - (1^2 + 2.2^2 + \ldots\ldots + 20^2)$

$= 20[(22 + 24 + \ldots\ldots + 60) + (24 + 28 + \ldots\ldots + 60)]$

$= 20\left[\frac{20}{2}(22+60) + \frac{10}{2}(24+60)\right]$

$100\lambda = 100 \times 248 \Rightarrow \lambda = 248$

27. (4)

$\sin x\, dy + y\cos x\, dx = 4x\, dx$

$\Rightarrow\ d(y\sin x) = 4x\, dx$

Integrating, $y\sin x = 2x^2 + c$

Curve passes through $\left(\frac{\pi}{2}, 0\right)$

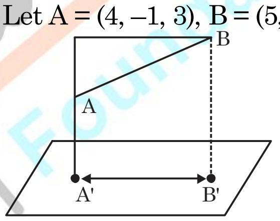
$\Rightarrow\ 0 = \frac{\pi^2}{2} + c \Rightarrow\ c = -\frac{\pi^2}{2}$

Now, $y\sin x = 2x^2 - \frac{\pi^2}{2}$

$\therefore\ y\left(\frac{\pi}{6}\right) = -\frac{8}{9}\pi^2$

28. (1)

Let A = (4, –1, 3), B = (5, –1, 4)



$AC = \overrightarrow{AB}\cdot\widehat{AC} = (\hat{i}+\hat{k})\cdot\left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{\frac{2}{3}}$

$\Rightarrow$ Length of projection $= \sqrt{\frac{2}{3}}$

29. (1)

Case – 1 :

$2\left(3-\sqrt{x}\right) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow\ x - 8\sqrt{x} + 12 = 0\ \Rightarrow\ \sqrt{x} = 4, 2$

$\Rightarrow$ x = 16 or x = 4 (Rejected)

Case – 2 :

Let $x \in [9, \infty)$

$2\left(\sqrt{x}-3\right) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow\ x - 4\sqrt{x} = 0\ \Rightarrow\ x = 16$ or $x = 0$ (Rejected)

So, x = 4, 16 are two solutions.

30. (4)

$$\sum_{k=0}^{12} a_{4k+1} = 416$$

$$\Rightarrow \frac{13}{2}(2a_1 + 48d) = 416$$

$$\Rightarrow a_1 + 24d = 31 \quad \text{...(i)}$$

$a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50_d = 66$ ...(ii)

from (i) and (ii), $d = 1$, $a_1 = 8$

Now, $140\ m = \sum_{r=1}^{17} ar^2$

$$= \sum_{r=1}^{17}\left[8 - 1(r-1)\cdot 1\right]^2$$

$$= \sum_{r=1}^{17}(r+7)^2$$

$$= \sum_{r=1}^{24} r^2 - \sum_{r=1}^{7} r^2$$

$$= \frac{24 \times 25 \times 49}{6} - \frac{7 \times 8 \times 15}{6}$$

$\Rightarrow \quad m = 34$